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Rationalized Speed/altitude Thresholds for ABM Testing

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The Antiballistic Missile (ABM) Treaty sharply restricts the development, testing, and deployment of defenses against strategic ballistic missiles. But it places no limitations on military systems for air defense, defense against tactical ballistic missiles, or the destruction of satellites. Without a clear definitional line between ABM defenses and other military systems, a nation that developed, tested, and deployed these other systems might be able to acquire a significant ABM capability despite the ABM treaty. Specific quantitative criteria that differentiate between permitted and prohibited activities could help to define such a line. This note proposes that any test involving a target with an altitude in excess of about 70 kilometers or a speed in excess of about 3 kilometers per second at the moment of closest approach between a weapon and a target should be considered either a "test in an ABM mode" or a test involving a target equivalent to a strategic ballistic missile or its elements in flight trajectory. The resulting clarification would provide a more objective standard of compliance and would also allow less leeway in the conduct of tests that are inconsistent with the purpose of the treaty.

GRAY AREAS OF THE ABM TREATY

One potential ambiguity in the ABM treaty arises because of the treaty's use of the phrase "strategic ballistic missile." The ABM treaty limits defenses against strategic ballistic missiles, which are understood by both sides to be those ballistic missiles that are able to attack targets of one side and that are

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Disclaimer: the views expressed in this note are those of the author alone, and do not necessarily represent those of any organization with which he is now or has ever been affiliated launched from bases located on the homeland of the other side or launched from submarines belonging to that other side. For missiles launched from homeland territory, missile range is a plausible way to distinguish strategic from nonstrategic ballistic missiles. But submarine-launched ballistic missiles (SLBMs) can be launched from any point in the ocean; even SLBMs of short range are able to attack homeland targets if launched near the coast. A problem arises if a nonstrategic ballistic missile (against which defenses are unconstrained) can have a range comparable to those of short-range SLBMs (against which defenses are limited).

A second, more difficult gray area is the overlap between ABM defenses and antisatellite (ASAT) weapons. The trajectory of a re-entry vehicle (RV) from a strategic ballistic missile can take it to speeds and altitudes comparable to those of satellites in low earth orbit. Therefore, it is clear that weapons intended to destroy satellites could have some potential against the RVs of strategic ballistic missiles.

The primary difference between satellite and missile targets is that the trajectory of a satellite, which orbits the earth in a highly predictable manner, is much better known than the trajectory of a ballistic missile, which becomes known only minutes before intercept. If trajectory data on a satellite are given to an ASAT weapon only minutes before intercept, and the ASAT were capable of operating on the same time scale, the ASAT could be operated in an environment that would strongly resemble a system tested in an ABM mode against an RV in midcourse.[1]

A third potential gray area relates to defenses against the ballistic missiles of third parties. Although the Intermediate Nuclear Forces (INF) Treaty of 1987 provides for the elimination of US and Soviet land-based ballistic missiles with ranges between 500 and 5,000 kilometers, other nations are not forbidden from possessing similar missiles with comparable ranges. Defenses against these third-party missiles could also have some capability against certain US or Soviet SLBMs, and could thus lead to compliance disputes.

A fourth gray area is that the ABM treaty forbids tests of non-ABM interceptors "in an ABM mode." At present, the US and the Soviet Union do not share a definition for this phrase that is based on a physical quantity such as altitude. However, during the ABM treaty negotiations, the US delegation did make a unilateral statement concerning the definition of the phrase "tested in an ABM mode." In part, it reads as follows:[2]

> To clarify our interpretation of "tested in an ABM mode", we note that we would consider a...missile...to be "tested in an ABM mode" if, for example,...an interceptor missile is flight-tested to an altitude inconsistent with interception of targets against which air defenses are deployed.

Such a statement clearly suggests the US negotiators' belief in 1972 that the altitude of a target is relevant to the definition of a "test in an ABM mode."

In addition, there is a historical precedent to use speed and altitude as parameters that might help to define a "strategic ballistic missile or its flight elements." The so-called "Foster box" was promulgated under US Undersecretary of Defense for Research and Engineering John Foster soon after the ABM treaty had been ratified. It was originally used to specify a speed–altitude boundary for targets within which US tests could be assumed to have nothing to do with strategic ballistic missiles or ABM defenses—providing a working definition for the US Department of Defense of what a test in an ABM mode is *not*. Any test involving objects falling outside the Foster box boundary was deemed worthy of more extensive internal review by the US government, and such tests were allowed or disallowed on a case-by-case basis.

The precise boundaries of the Foster box are classified, but figure 1 illustrates a hypothetical box with thresholds of 3 kilometers per second speed and 70 kilometers altitude.

The negotiated use of a Foster-like box to define permitted and prohibited tests would help to prevent the exploitation of the gray areas involving nonstrategic versus strategic ballistic missiles and satellites versus strategic ballistic missiles.

THE MODEL

This paper proposes that target speed and altitude be used as parameters to make definitions for "strategic ballistic missile" and "test in an ABM mode" more clear. In particular, it is proposed that any test involving a target with an altitude in excess of about 70 kilometers or a speed in excess of about 3 kilometers per second at the moment of closest approach should be considered either a "test in an ABM mode" or a test involving a target equivalent to a strategic ballistic missile.[3] This proposal defines what a test in an ABM mode *is*. In so doing, it is more restrictive than the Foster box, which was used to define what a test in an ABM mode is *not*.

The model used to justify these threshold values assumes a target ballistic missile minimum-energy ballistic trajectory in a vacuum over a nonrotating spherical earth for the portion of the trajectory from launch at the earth's surface (where the acceleration to ballistic velocity is assumed to be instantaneous), until the target descends to an altitude of 100 kilometers after passing through apogee. From this 100-kilometer altitude until impact, a flat earth with atmospheric drag is assumed.

The mathematical details of the model are presented in the appendix, and are used to calculate the plots presented in figures 2 and 3.

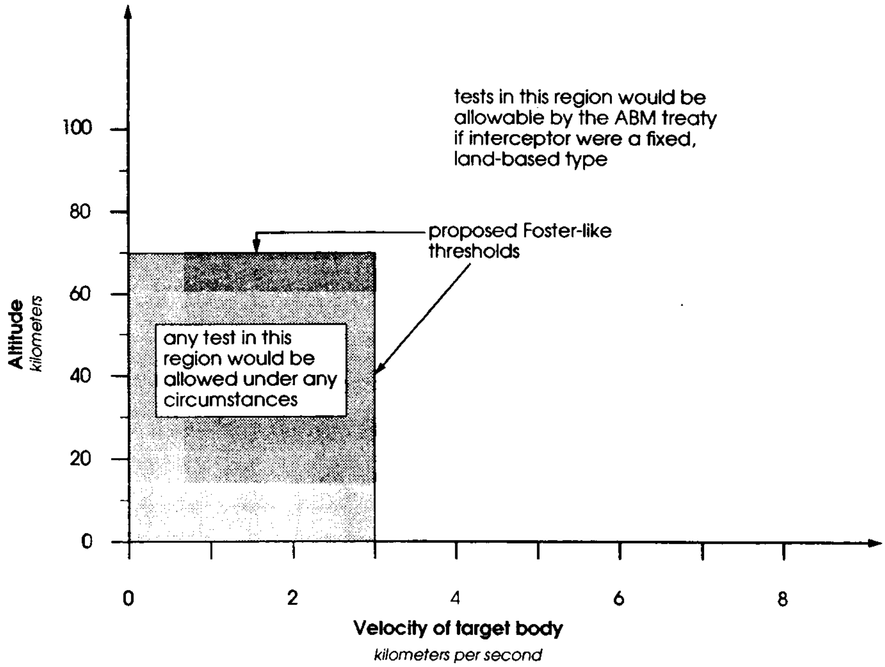

Figure 1: Hypothetical Foster box with thresholds of 3 kilometers per second and 70 kilometers

ANALYSIS

Figure 2 plots speed–altitude relationships for re-entry vehicles on ballistic minimum-energy trajectories of various ranges and plausible ballistic coefficients, as they approach their points of impact.[4] For convenience, the proposed thresholds of 70 kilometers and 3 kilometers per second are reproduced on this figure.

According to the SALT II treaty definition, the SS-N-6 SLBM, with a range of 2,500 kilometers, is a strategic ballistic missile; no strategic ballistic missile has a shorter range. A missile of 2,500 kilometers maximum range could reach top speeds somewhat in excess of 4 kilometers per second. Defenses capable of intercepting targets with speeds of 4 kilometers per second or larger are therefore capable of intercepting strategic ballistic missiles.[5]

By contrast, a ballistic missile of 900 kilometers range has a maximum speed of somewhat under 3 kilometers per second. At least one tactical

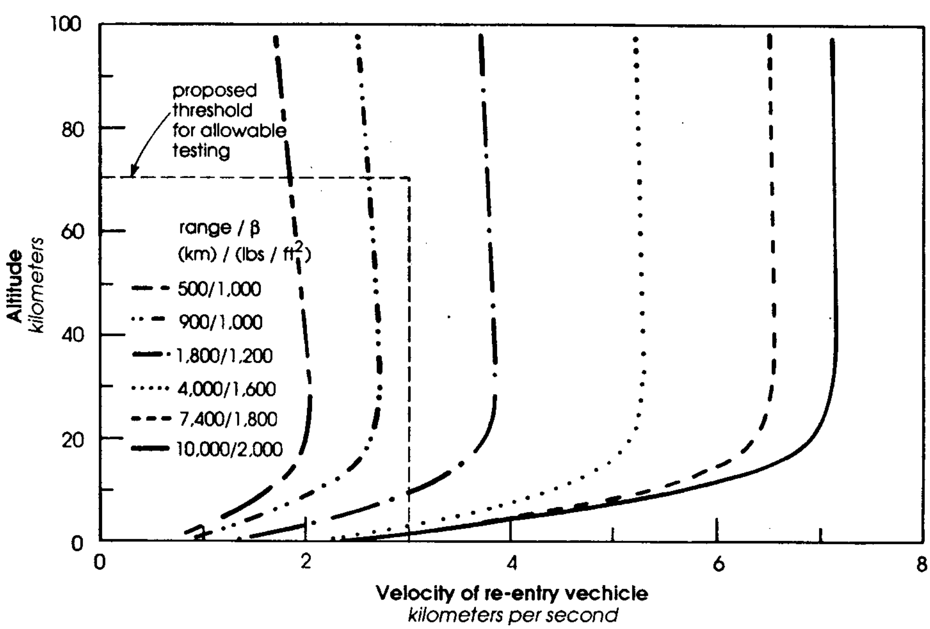

Figure 2: Speed–altitude relationships for missiles of various ranges and ballistic coefficients

ballistic missile—the Soviet SS-12 being retired under the INF treaty—has such a range. The ABM treaty was not intended to constrain defenses against such missiles.

We conclude that a plausible distinction in speed between strategic and nonstrategic ballistic missiles for the purposes of the ABM treaty lies somewhere in the range between somewhat under 3 kilometers per second and somewhat above 4 kilometers per second. Adoption of the lower end of this range (3 kilometers per second) would be consistent with a philosophy of treaty compliance that did not endorse the exploitation of gray areas.

An altitude threshold is more difficult to set appropriately. Defenses against high-flying aircraft are clearly not prohibited by the ABM treaty, and thus the altitude at which reconnaissance airplanes such as the US SR-71 have flown (about 30 kilometers) set a lower bound for an altitude threshold.

Another way to set an altitude threshold is to set it well above the maximum altitude at which modern surface-to-air missiles (SAMs) are lethal. This would then allow tests of currently deployed SAMs, at least against slower (tactical) ballistic missiles, and avoid the unnecessary complication of prohibiting tests against *non*strategic missiles by an already deployed system.

The range of modern SAMs is on the order of 100 kilometers.[6] However, few sources are specific about whether this refers to range as measured on the ground (i.e. ground range) or as measured from launch point to impact point (i.e. slant range). I will assume that a SAM with a "range of 100 kilometers" means that it has a slant range of 100 kilometers in all directions, i.e. that its lethal envelope is a hemisphere of radius 100 kilometers. This is a simplifying assumption to which I will return shortly.

Figure 3 illustrates trajectories near the point of impact for several ballistic missiles on minimum-energy trajectories. The uppermost trajectory is that of a missile with a maximum range of 900 kilometers. When this missile is about 100 kilometers from impact, its altitude is about 65 kilometers. Therefore, to allow a SAM with a slant range of 100 kilometers launched from the ballistic missile's intended point of impact to be tested against such a tactical missile, one would have to place the altitude cap on the Foster box above 65 kilometers.

Note, however, that the assumption of a hemisphere of 100-kilometer lethal radius overstates the SAM's capability by exaggerating the altitude that it could reach. The highest attainable altitude for the SAM would be reached if it were fired straight up, and if the ground range was 100 kilometers, this maximum altitude would be closer to 50 kilometers than to 100. The actual lethal envelope is a flattened hemisphere with an altitude perhaps half its ground range. Thus, even if the direction of the incoming missile were known and the SAM located upstream of the impact point, the use of 70 kilometers as an altitude threshold (as opposed to 30-40 kilometers) is sufficient to provide "head room" for currently deployed SAMs (or antitactical ballistic missiles that may be developed in the future) to be tested at their full range without violating such a Foster box's altitude threshold.

DISCUSSION AND CONCLUSIONS

This analysis suggests that thresholds of 3 kilometers per second in speed and 70 kilometers in altitude could be used to define a "test in an ABM mode" or

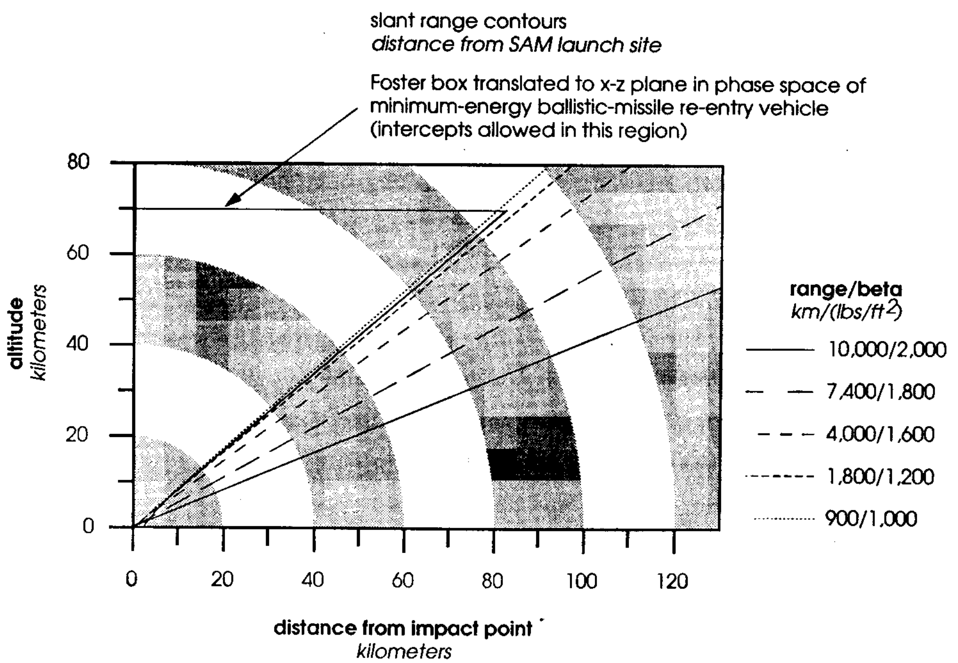

Figure 3: Trajectories near the point of impact for several ballistic missiles

a "strategic ballistic missile" for purposes of the ABM treaty. Any test involving a target with a speed below 3 kilometers per second and an altitude below 70 kilometers at the moment of closest approach between interceptor and target would be allowed under all circumstances. A fixed land-based interceptor could be tested against a target faster than 3 kilometers per second or higher than 70 kilometers in accord with the development and testing of fixed land-based ABM interceptors allowed by the ABM treaty.

The US–Soviet Agreed Statement of 1978 defining tests in an ABM mode states in part that:[7]

> an interceptor missile is considered to be "tested in an ABM mode" if it has attempted to intercept (successfully or not) a strategic ballistic missile or its elements in flight trajectory...."Strategic ballistic missiles or their elements in flight trajectory" include ballistic target-missiles with the flight trajectory characteristics of strategic ballistic missiles or their elements over the portions of the flight trajectory involved in testing.

According to the proposal made in this paper, this text would be replaced by something along the following lines:

> an interceptor missile is considered to be "tested in an ABM mode" if it has attempted to intercept (successfully or not) an object whose velocity exceeds 3 kilometers per second or whose altitude exceeds 70 kilometers at the moment of closest approach between interceptor and object.

A similar understanding would be reached regarding the test of a radar in an ABM mode. In particular, the Agreed Statement of 1978 reportedly specifies that a radar is considered to be tested in an ABM mode if it performs certain functions such as[8]

- tracking and guiding an ABM interceptor missile; or
- tracking strategic ballistic missiles or their elements in flight trajectory in conjunction with an ABM radar which is tracking and guiding an ABM interceptor missile.

This text would be replaced by something like:

- tracking and guiding an ABM interceptor missile or an interceptor missile tested in an ABM mode; or

- tracking an object whose velocity exceeds 3 kilometers per second or whose altitude exceeds 70 kilometers during radar illumination in conjunction with an ABM radar that is tracking and guiding an ABM interceptor missile or an interceptor missile being tested in an ABM mode.

It will be noted that these modified Agreed Statements would prohibit testing against satellites of all interceptors that are not of the fixed land-based variety. Thus, these modified Agreed Statements are part of an arms control regime that goes beyond the current ABM treaty. A clause that provided an exception for objects in orbit around the earth could be added if the two sides wished to preserve an option for developing and testing ASAT weapons that are mobile.

Additional issues would have to be resolved through negotiation. For example, taken literally, the set of modified Agreed Statements might be construed to forbid docking by a space shuttle at a space station. This problem could be resolved by an agreement to permit all "intercepts" with a relative speed of less than 50 meters per second between target and interceptor when the distance between target and interceptor is less than 1 kilometer. (Fifty meters per second is somewhat arbitrary but is a value that is much smaller than any plausible closing velocity between a real interceptor and its target.)

A second issue is that tests of mechanisms designed for boost-phase interception might be "legal" under this regime. In the earliest stages of flight, ballistic missiles travel slowly while at low altitudes. Certain boost-phase weapons (such as ground-based lasers or air-launched interceptors) could in principle be tested against boosters in flight shortly after launch without violating speed-altitude threshold limitations. If both sides wanted to foreclose tests of weapons for boost-phase intercept, they could agree explicitly to forbid tests against missiles during boost-phase but within the speed-altitude thresholds.

A third issue is one of verification. The term "moment of closest approach" is well-defined in a physical sense, but cooperative measures might be necessary to verify this definition in practice.

In the regime proposed above, thresholds are defined with respect to the moment of closest approach between interceptor and target. However, a variant on the above regime would be to define the thresholds in terms of the greatest speed and altitude of the target in its entire trajectory (or would-be trajectory if it were intercepted), as was reportedly done with the original Foster box. This would make the regime much more stringent, since the apogee (rather than the target altitude at the moment of closest interceptor approach) would have to fit within the altitude threshold.

Finally, the two sides could simply agree to forbid all tests outside the 3 kilometers per second and 70 kilometers thresholds. This would be tantamount to eliminating the special status that the ABM treaty accords to fixed land-based interceptors, and would also de facto prohibit all ASAT tests. This would strengthen the ABM treaty regime by preventing ABM work disguised as ASAT work, and would help to preserve space assets on both sides.

To summarize, the possible regimes are characterized by four degrees of freedom: whether speed-altitude thresholds are applied to the moment of closest approach between target and interceptor or to the target at all points along its destined trajectory; whether these thresholds should also be applied to fixed land-based interceptors; whether boost-phase intercepts are allowed; and whether explicit exceptions for tests against satellites are allowed. In all cases, rules should be negotiated to permit nondestructive docking.

The argument has been made for a threshold in speed of 3 kilometers per second and a threshold in altitude of 70 kilometers at a certain moment in the trajectory of the target. But the important issue is not these precise figures, but rather that greater conceptual clarity can result from such an approach to resolving ambiguities.

Is greater conceptual clarity a plus or a minus? At present, compliance assessments are hampered by the lack of precise definitions. The result is that compliance assessments are now based on highly subjective judgments of whether or not a given piece of hardware could substitute fully for an ABM system or component, or whether or not a component is tested against a target with the flight characteristics of a strategic ballistic missile. A regime that complemented subjective judgments with more objective criteria would be very helpful from the perspective of those who endorse the central premise of the ABM treaty—that restrictions on the development and testing of ABM systems serves the interests of both the US and the Soviet Union.

Thus, the real issue is whether or not the US and the Soviet Union have the political will and desire to conduct their testing under negotiated restrictions. If they do, then the general approach described here—the specification of quantitative thresholds tied to system performance parameters beyond which testing is forbidden—may have some potential for clarifying the existing treaty regime. If they do not, neither this approach nor any other will prove workable.

Appendix

BALLISTIC-MISSILE KINEMATICS

We begin with the well-known equation defining the possible trajectories for projectile motion in a central gravitational force field in a vacuum.[9]

$$\frac{1}{r} = \frac{GM_e}{l^2}\left(1 + \left[1 + \frac{2El^2}{(GM_e)^2}\right]^{1/2}\cos(\theta + \theta_0)\right) \tag{1}$$

In the general case, this equation describes an ellipse with one focal point at the center of the earth and whose longitudinal axis is oriented at an angle $-\theta_0$.

In this equation

M_e is the mass of the earth, and G the universal gravitational constant;

r is the distance of the projectile from the center of the earth;

E is the total energy per unit mass of the projectile, which is constant and given by the sum of its kinetic energy $v_0^2/2$ and potential energy $-GM_e/a$ (each per unit mass) at the moment of launch, where v_0 is the initial speed of the projectile and a is the radius of the earth;

l is the angular momentum of the projectile per unit mass, also a constant, and given by $l = v_0 a \sin\gamma$, where γ is the angle that the velocity vector makes with the local vertical at the moment of launch.

If we take θ to be the angle of the projectile as measured counterclockwise from the place of launch, then θ_0 is given by:

$$\theta_0 = \cos^{-1}\left(\frac{\frac{l^2}{aGM_e} - 1}{\left[1 + \frac{2El^2}{(GM_e)^2}\right]^{1/2}}\right) \tag{2}$$

Figure 4 illustrates the angles and distances involved.

Equations 1 and 2 can combined and rewritten together (after a great deal of algebraic manipulation) to give a relationship between r, the earth's radius a, and the initial velocity and launch angle as:[10]

$$\frac{a}{r} = \frac{1 - \cos\theta}{\left(\frac{av_0^2}{GM_e}\right)\sin^2\gamma} + \frac{\sin(\gamma - \theta)}{\sin\gamma} \tag{3}$$

Note that in this equation r must be equal to a when $\theta = 0$ or, at the other end of the trajectory, when $\theta = R/a$, where R is the range of the projectile. The minimum-energy trajectory of a given range is specified by the launch angle γ that minimizes the projectile velocity needed to travel that range. We find that the angle γ_{min} that corresponds to the minimum necessary velocity is:

$$\gamma_{min} = \frac{1}{4}\left(\frac{R}{a} + \pi\right) \tag{4}$$

Equations 3 and 4 suffice to specify completely the minimum energy trajectory of a specified range. The conservation of energy also provides a relationship between the speed of the projectile and its radial position:

$$\frac{1}{2}v_0^2 - \frac{GM_e}{a} = \frac{1}{2}v^2 - \frac{GM_e}{r} \tag{5}$$

These three equations allow us to calculate the speed and angle of the projectile at an altitude of 100 kilometers, i.e., at point A in figure 4. By assumption, point A is the point at which the projectile enters the atmosphere, and atmospheric drag must now be taken into account. Below 100 kilometers, we use a modified exponential atmosphere, in which atmospheric density ρ is given by:[11]

$$\rho = \rho_0 \exp(-z/l) \tag{6}$$

where

z = height above the surface of the earth in kilofeet (0.305 kilometers)

The sea-level density $\rho_0 = 1.225$ kg m^{-3}

$l = 23 - 0.0164(z - 197) + 4.61\cos(0.02992[z - 197])$

To calculate the velocity as a function of altitude, we employ a flat earth approximation, in which Newton's Second Law gives:

$$\begin{aligned} \frac{dv_x}{dt} &= -\frac{\rho v^2}{2\beta}\left(\frac{v_x}{v}\right) \\ \frac{dv_z}{dt} &= -g - \frac{\rho v^2}{2\beta}\left(\frac{v_z}{v}\right) \end{aligned} \tag{7}$$

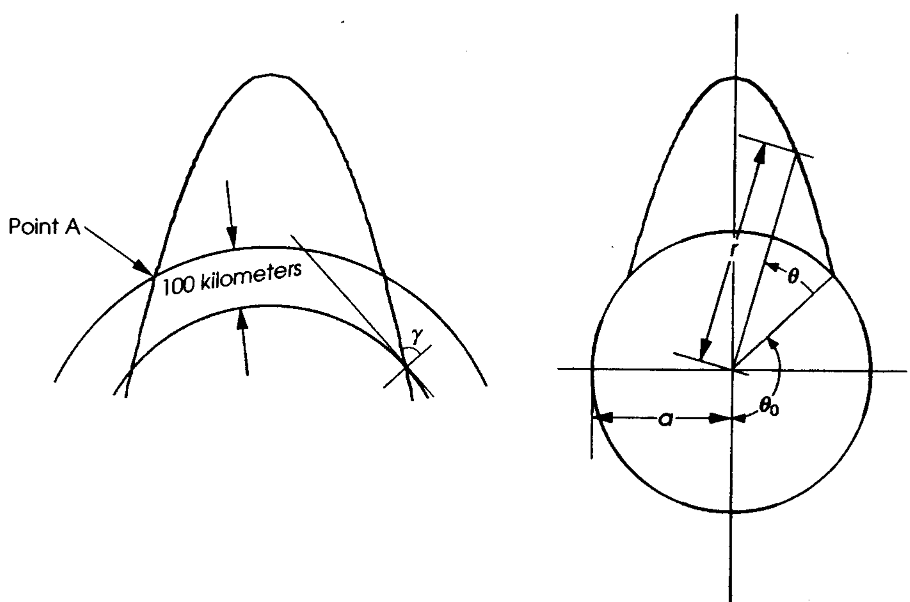

Figure 4: Idealized ballistic missile trajectories on a spherical earth

where x is the direction of the horizontal motion along the trajectory and β is the ratio of the mass of the projectile to its drag area. (A point projectile is assumed, with the consequence that β is the only parameter that affects the motion of the RV. The dependence of the resisting force on the square of the velocity arises from an assumption that the only force acting on the projectile comes from the deflection of the airstream against the moving projectile.[12]) Modern RVs have β in the region of 2,000 lbs per square foot (about 10,000 kg m^{-2}), old RVs perhaps a few to several hundred pounds per square foot.

These equations can be numerically integrated to provide the relationships between v_x and v_z as functions of altitude, shown in figure 2. Defining v as $(v_x^2 + v_z^2)^{1/2}$ provides a relation between v and z, characterized by β and the range of the projectile.

Once v_x has been obtained as a function of altitude, it can be integrated numerically once more, resulting in an x–z plot of the trajectory itself in the presence of atmospheric drag.

NOTES AND REFERENCES

1. For further discussion on this point, see Ashton Carter, "The Relationship of ASAT and BMD Systems," *Daedalus*, Spring 1985.

2. US Arms Control and Disarmament Agency, *Arms Control and Disarmament Agreements*, August 1980, pp.146–147.

3. These threshold values for speed and altitude were previously proposed in Herbert Lin, *New Weapon Technologies and the ABM Treaty* (Pergamon-Brassey's International Defense Publishers, 1988). This book also addresses in greater detail many of the conclusions presented here.

4. For example, it has been reported that modern US re-entry vehicles have ballistic coefficients around 2,000 lbs ft^{-2}. See Matthew Bunn and Kosta Tsipis, "Ballistic Missile Guidance and Technical Uncertainties of Countersilo Attacks," Report 9, Program in Science and Technology for International Security, Massachusetts Institute of Technology, August 1983, note R7.

5. As the Soviet Union begins to retire short-range strategic ballistic missiles such as the SS-N-6, the characteristic maximum speeds will increase significantly. Only 12 Soviet SS-N-17 SLBMs are deployed (maximum range of 3,900 kilometers) The US Poseidon (also being retired) has a maximum range of 4,600 kilometers. All other US and Soviet SLBMs have maximum ranges greater than 6,500 kilometers.

6. *Soviet Military Power 1985* lists the Soviet SA-10 and SA-12 with ranges of 100 kilometers and the US Patriot with a range of more than 80 kilometers. See p.53.

7. Report to the Congress on the SDI, June 1986, p.C-7.

8. Ibid.

9. Cf. Herbert Goldstein, *Classical Mechanics* (Addison-Wesley Publishing Company, 1950), p.77, equation 3-46.

10. Albert D. Wheelon, "Free Flight of a Ballistic Missile," *ARS Journal*, December 1959, p.915.

11. The atmospheric model used is from John Martin, *Atmospheric Reentry* (Prentice-Hall, 1966), p.239.

12. For more discussion, see L. Prandtl and O.G. Tietjens, *Applied Hydro- and Aeromechanics* (Dover Publications, 1934), chapter 5.