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The Gravity Gradiometer as a Verification Tool

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A gravity gradiometer is an incredibly sensitive device whose unit of measure is an eotvos (10^{-9} s^{-2}), which is approximately equal to the gradient of the gravitational field produced by 10 grains (about 7.5 milligrams) of sand at a distance of 1 centimeter. We consider the gradiometer engineered by Draper Laboratories.[1] Such a device could be used in portal-perimeter monitoring to detect gravitational effects produced by the mass distributions of shrouded containers or the contents of railcars and trucks moving through the portals or in observations of missiles to determine the number of warheads they carry or whether extraordinary shielding had been emplaced to block telltale emissions of nuclear radiation. It could also be used in some circumstances to distinguish non-nuclear cruise missiles from cruise missiles carrying nuclear weapons if measurements that used neutron or gamma-ray irradiation were considered too intrusive.

THE GRAVITY GRADIOMETER is a largely unfamiliar device that could have significant value as a verification tool. The device and its applications are summarized in this paper.

OPERATING PRINCIPLE

The basic operating principle of the device is actually quite simple.[2] The main sensing element is a metallic sphere with high-density weights located at opposite ends of a diameter. The sensing sphere or float is symmetric about this diameter. It is suspended both electrostatically and by a fluid within a hollow sphere and is free to rotate inside. When a given mass is passed by the device the mass pulls more on the nearer weight than on the farther. A torque

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is thereby exerted on the sensing sphere, and the device then exerts a counter torque just sufficient to stop the motion of the sphere. The amount of counter torque is determined by a feedback system that is calibrated to a source of electric current. Since the moments of inertia of the sensing sphere are precisely known and the counter torque is measured, the components of the gradient of the gravitational field can be inferred.

As the torques being measured are extremely small, it is rather surprising that effects associated with vibrations produced by the surrounding environment and thermal effects do not mask the measurement. However, for a specific arrangement of three gradiometers, angular jitter can be accurately measured since there are no torques acting around each gradiometer's axis of symmetry—the diameter through the weights. Three such axes provide three components of the angular displacement vector that completely determines the angular jitter. Therefore if a system of three gradiometers is placed inside a room that is vibrating, angular jitter can be accurately measured and subtracted from the measurement. The three-gradiometer version has not been thoroughly tested, but the developers have high expectations for its performance.[3]

The weight of the complete Draper device, including three gradiometers, gimbals, electronics, computers, and power supply, is estimated to be about 230 kilograms. It could be transported easily on a pallet.*

Thermally caused imbalances have been eliminated from the device by placing it in a hard vacuum, utilizing a balanced thermal Wheatstone bridge, and surrounding it by ample amounts of gold plating (polished gold reflects about 99 percent of thermal infrared radiation). Internal temperatures are controlled to within one millionth of a degree fahrenheit. Once the device has been stabilized (i.e. is in thermal equilibrium) it can be transported and immediately used. The response time of the gradiometer is set by a filter, which is chosen according to the amount of noise present. A typical response

* Bell Aerospace has developed an entirely different design of gradiometer. It employs four accelerometers equally spaced on a rotating platform. The central unit and the accompanying electronics package for data analysis can be designed to weigh about 23 kilograms. Comparable levels of accuracy can be obtained with the Bell gradiometer. Private communication, Ernest H. Metzger, executive director of engineering, inertial products, Bell Aerospace, Syracuse, New York, March 1989.

time of 1 minute for a signal of approximately 14 eotvos units has been demonstrated for a single gradiometer with 0.25–eotvos-unit accuracy through a measurement of the gravity gradient produced by an 8.75-pound (4 kilogram) lead ball.[4] The response time can be shortened by changing the filter, but there is an additional noise penalty. The noise level is in fact a function of the response time, which for short response times (less than 30 seconds) varies as the inverse square of the response time (mostly due to jitter) and for long response times varies as the inverse of the square root of the response time (Brownian motion). For a three-gradiometer device with an accuracy of 1 eotvos unit, the response time could be as short as 10 seconds for a reasonable signal-to-noise ratio.

In practical applications, both the effects associated with the surrounding environment and internal device imperfections (for example, separation of the center of gravity and the center of flotation of the sensing sphere) set a device bias that is essentially constant over time. Actual measurements are therefore made relative to this bias. This does not affect measurement accuracy.

A properly packaged and field-tested three-gradiometer device would cost about 2 million dollars. One could imagine a three-gradiometer device placed on a tripod,[5] and the object whose gravitational gradients were to be scanned—for example, a missile—would pass by the device at an appropriate rate. It might be desirable to allow for several different scans along the object length at different distances from its axis. Alternatively, the object could be stationary while the gradiometer passed by. Since the response time of the device is relatively short, the time required to scan the object would not be a significant factor in the monitoring process. Furthermore the resolution of the device—its capability of discerning details about the individual components of the object—would be easy to control by limiting how close to the object the gradiometer could approach.

MEASUREMENT OF A CRUISE MISSILE

As an illustrative example, let us consider the use of a gradiometer in cruise-missile verification. We can roughly estimate the minimum measurement distance required to use a gradiometer to distinguish a conventional from a nuclear cruise missile of a Tomahawk type[6] by first determining the required level of accuracy for a measurement of one of the gradients.

Assume for the moment that a cruise missile may be approximated as an infinite cylinder of uniform mass density. A fractional inaccuracy in a measurement of the radial gradient of the gravitational field F' would translate into an uncertainty in the determination of the mass density ρ:

$$\frac{\delta F'}{F'} = \frac{\delta \rho}{\rho} \tag{1}$$

where $\delta F'$ is the uncertainty in the radial gradient and $\delta\rho$ the uncertainty in the mass density. Using the analytic form of the field produced by a uniform cylinder of infinite length equation 1 becomes

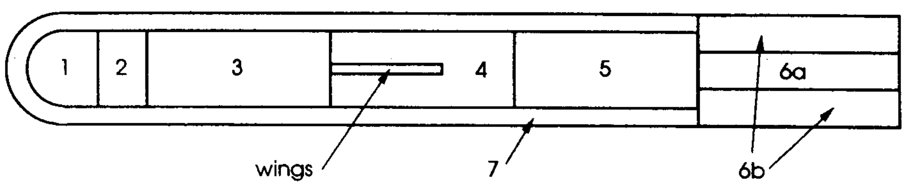
$$\frac{\delta \rho}{\rho} = \frac{r^2 \delta F'}{2\pi a^2 \rho G} \tag{2}$$

where a is the radius of the cylinder, r is the radius at which the measurement is to be made, and G the gravitational constant ($6.67{\cdot}10^{-11}$ N m^2 kg^{-2}. Gradiometer technology can achieve levels of accuracy down to the 1–eotvos-unit level or 10^{-9} s^{-2}, therefore we assume $\delta F' \approx 1$ eotvos unit. We also assume $a = 0.265$ meters (the Tomahawk radius) and $\rho = 2{\cdot}10^3$ kg m^{-3}, between that of jet fuel and aluminum. If we require 2-percent accuracy, equation 2 gives $r \approx 1$ meter, which is roughly twice the diameter of the Tomahawk.

From the point of view of the gravity gradiometer, the main differences between the conventional and nuclear versions of the Tomahawk are the mass density, length, and locations of their warheads. The nuclear warhead in the US Tomahawk is more than twice as dense, half as long, and more forward in location than the conventional warhead. More than half the volume of the fuel in the nuclear version is in roughly the same location as the warhead in the conventional version; however the density of the fuel is roughly 20 percent less than that of the conventional warhead. Since the gravitational effects of mass cannot be shielded, if the gradiometer could approach within a meter or so of the missile, C-SLCMs should be easily distinguished from N-SLCMs.

In order to simulate a scan of a cruise missile with a gradiometer, models of the internal structure of the nuclear and conventional versions were constructed. These models are shown in figures 1 and 2.[7] They are derived from Tsipis[8] with some minor modifications. The real test of the practical value of the gradiometer would require actual cruise-missile design data.

In these calculations the aluminum skin of the hemispherical nose and its volume mass density are approximated by points located at their respective centers of mass. The remaining components including the airframe are treated as lines of mass along the axis of the missile. The computation of the various gradients of the gravitational field produced by this mass distribution are discussed in appendix 1.[9]



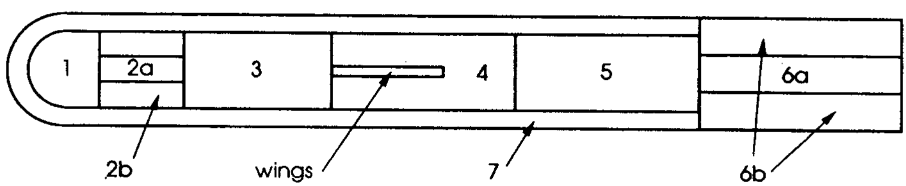
Section	Component	Mass *kilograms*	Length *meters*	Radius *meters*	Skin thickness *meters*	Average density *g cm^{-3}*
1*	Guidance system	68	0.65	0.25	0.013	0.61
2	Fuel	27	0.13	0.25	0.013	1.07
3	Warhead	456	1.77	0.25	0.013	1.30
4†	Fuel	176	1.40	0.24	0.021	0.67
5	Engine	59	1.64	0.25	0.013	0.18
6a	Fuel	178	0.65	0.22	0.000	1.80
6b	Rocket	121	0.65	0.27	0.000	2.70
7	Skin or airframe	365	5.59			2.70

* Nose is assumed to be a hemisphere of radius 0.25 meters.

† Wings are included in airframe which accounts for smaller inner radius. Top surface of wings is assumed to have an area of 1.02 square meters and the mass of the wings is assumed to be 52.5 kilograms.

Figure 1: A model sectional representation of a conventionally armed Tomahawk SLCM, giving numerical values for physical characteristics of each longitudinal section. The radius corresponds to the outer boundary of a section and the skin thickness varies from section to section as indicated.

The results of the calculations are depicted in figure 3, which shows a comparison of simulated measurements of the radial gradient of the radial component of the gravitational field, F_{rr}, produced by a conventional and nuclear cruise missile along their length with a gradiometer located at 1 meter from the missile's axis in figure 3a and at 0.5 meters in figure 3b. The error bars in figures 3a and 3b are realistic. According to figure 4a, the difference between a conventional and nuclear version at 1 meter is measurable. If the measurement is made at closer range as in figure 3b the difference between the two measurements is even greater. For the cruise-missile models we have employed, the gradiometer is sensitive to the mass per unit length along the cruise missile. The nuclear warhead and the surrounding fuel have the largest mass per unit length. This feature coupled with the size of the



Section	Component	Mass *kilograms*	Length *meters*	Radius *meters*	Skin thickness *meters*	Average density *g cm*$^{-3}$
1*	Guidance system	46	0.46	0.25	0.013	0.61
2a	Warhead	123	0.87	0.13	0.000	2.70
2b	Fuel	123	0.87	0.25	0.013	0.97
3	Fuel	260	1.22	0.25	0.013	1.07
4†	Fuel	176	1.40	0.24	0.021	0.67
5	Engine	59	1.64	0.25	0.013	0.18
6a	Fuel	178	0.65	0.22	0.000	1.80
6b	Rocket	121	0.65	0.27	0.000	2.70
7	Skin or Airframe	365	5.59			2.70

* Nose is assumed to be a hemisphere of radius 0.25 meters.

† Wings are included in airframe which accounts for smaller inner radius. Top surface of wings is assumed to have an area of 1.02 square meters and the mass of the wings is assumed to be 52.5 kilograms.

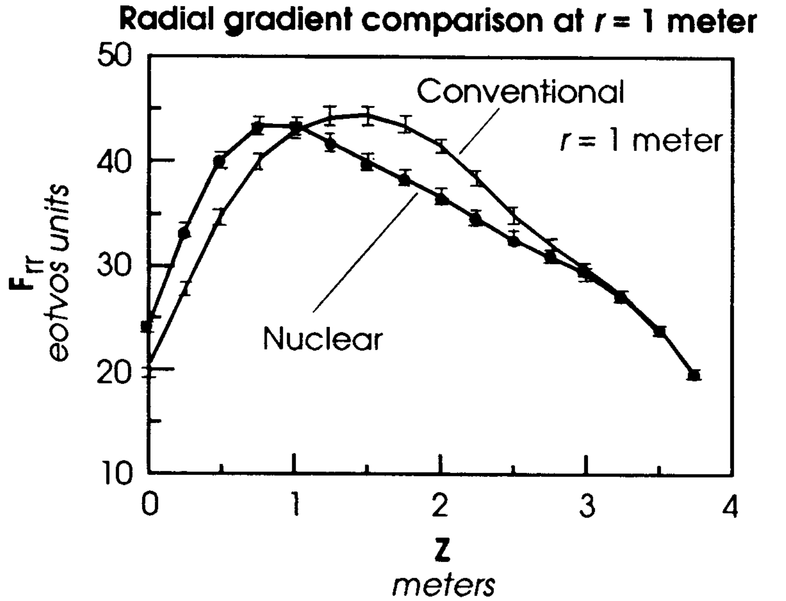
Figure 2: A model sectional representation of a nuclear armed Tomahawk SLCM, giving numerical values for physical characteristics of each longitudinal section. The radius corresponds to the outer boundary of a section and the skin thickness varies from section to section as indicated.

warhead make these components discernible with the gradiometer scan. Figures 4a and 4b show that the difference between a conventional and nuclear cruise missile is also measurable when the longitudinal gradient (the rate of change along the length of the cruise missile) of the radial component of the gravitational field F_{rz} is measured along the length of the cruise missile.

If measurements are not made continuously along the missile axis, then the stepsize (the distance between measurements and the distance from the object) must be less than the size of an object in order to resolve it. This is shown in figures 5a, 5b, and 5c. Each of these figures depicts a comparison between a simulated scan of a 250-kilogram point mass located 1.5 meters from the origin and a scan of two point masses each of 125 kilograms and located at 1.25 meters and 1.75 meters respectively—0.5 meters apart. The stepsize was chosen to be 0.2 meters in figures 5a and 5b. In figure 5a the radial distance is 2 meters so the two curves are essentially identical. However, as figure 5b reveals, the two masses can easily be resolved at a radial distance of 0.4 meters. But figure 5c demonstrates that for a stepsize of 0.5 meters and a radial distance of 0.4 meters the two objects cannot be resolved. Since the nuclear warhead is roughly a meter in length, a stepsize of 0.25 meters should be sufficient.

The closer the gradiometer is to the missile the more accurate the measurement. Twelve measurements, each involving a signal of 100 eotvos units, individually spaced by 0.25 meters, and made with a single gradiometer with 1 eotvos unit of accuracy, would take a total measurement time of 12 minutes. If the signal-to-noise ratio were kept fixed at 56 (the ratio used in the 8.75-pound lead ball measurement alluded to above), then the total measurement time could be as low as 4.5 minutes. For the three-gradiometer device with 1 eotvos unit of accuracy the total measurement time could be as low as 2 minutes and even less if higher noise levels are acceptable.

The possibility that a gradiometer scan could be spoofed cannot of course be discounted. Spoofing would be difficult, however, because of the sensitivity of existing gradiometers and the fact that the gradients of the gravitational field depend upon all the mass moments, i.e. total mass, center of mass, moments of inertia, and higher moments. Trying to arrange for all five independent gradients to be the same is roughly equivalent to trying to arrange for all the mass moments to be the same.



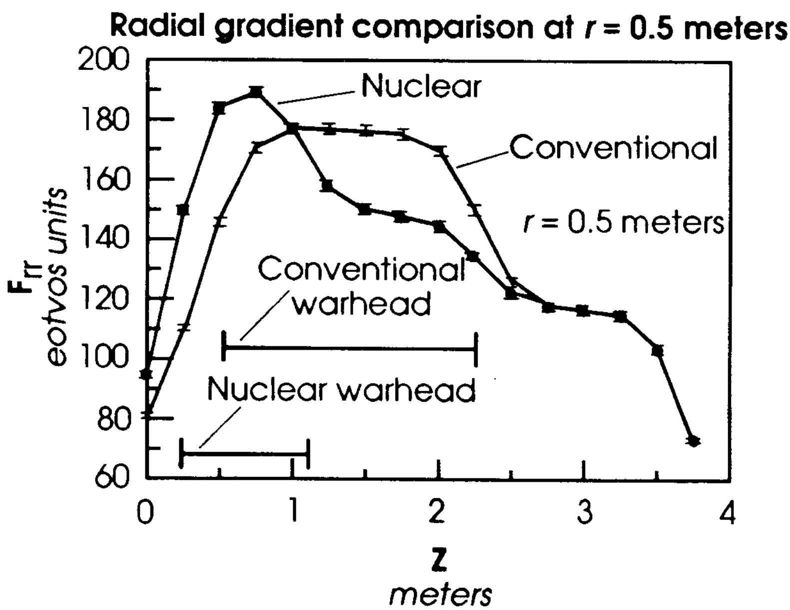
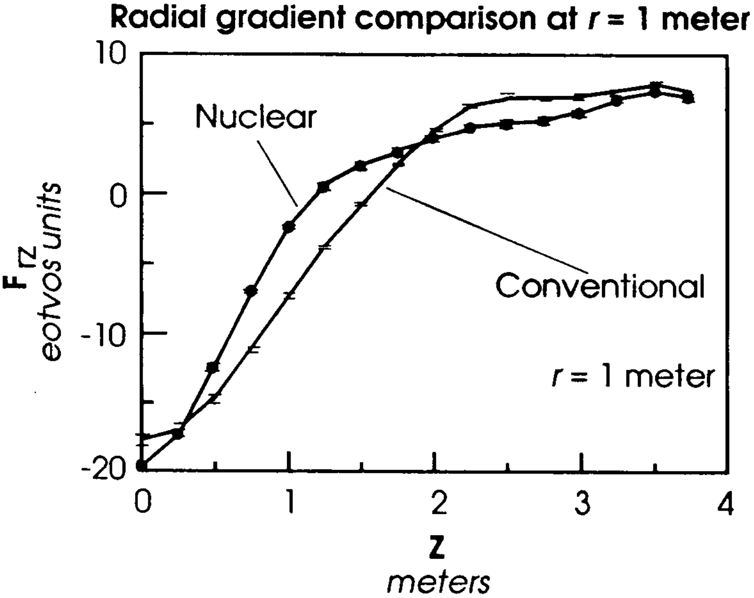


Figure 3: A comparison of the radial gradient of the radial component of the gravitational field F_{rr} for nuclear and conventional cruise missiles as calculated from the models depicted in figures 1 and 2. The radial distances *r* from the missile axis are 1 meter in figure 3a and 0.5 meters in 3b. *Z* measures the distance along the cruise missile axis. The origin, $Z = 0$, coincides with the base plane of the hemispherical nose of a cruise missile. The error bars are realistic (about 1 eotvos unit) and correspond to a 2-percent error in 3a and a 1-percent error in 3b.



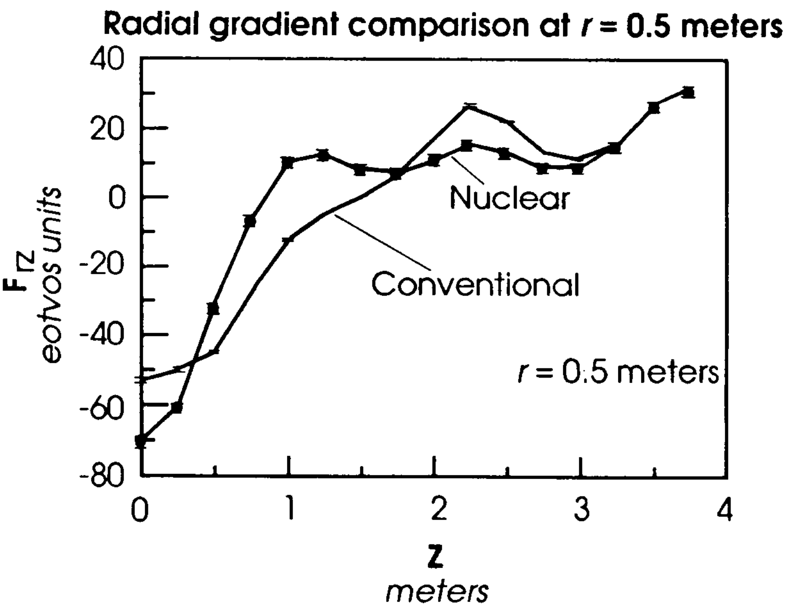


Figure 4: A comparison of the radial gradient of the axial component of the gravitational field F_{rz} for nuclear and conventional cruise missiles as calculated from the models depicted in figures 1 and 2. The radial distances *r* from the missile axis are 1 meter in figure 4a and 0.5 meters in 4b. *Z* measures the distance along the cruise missile axis. The origin, *Z* = 0, coincides with the base plane of the hemispherical nose of a cruise missile. The error bars are realistic (about 1 eotvos unit) and correspond to a 2-percent error in 4a and a 1-percent error in 4b.

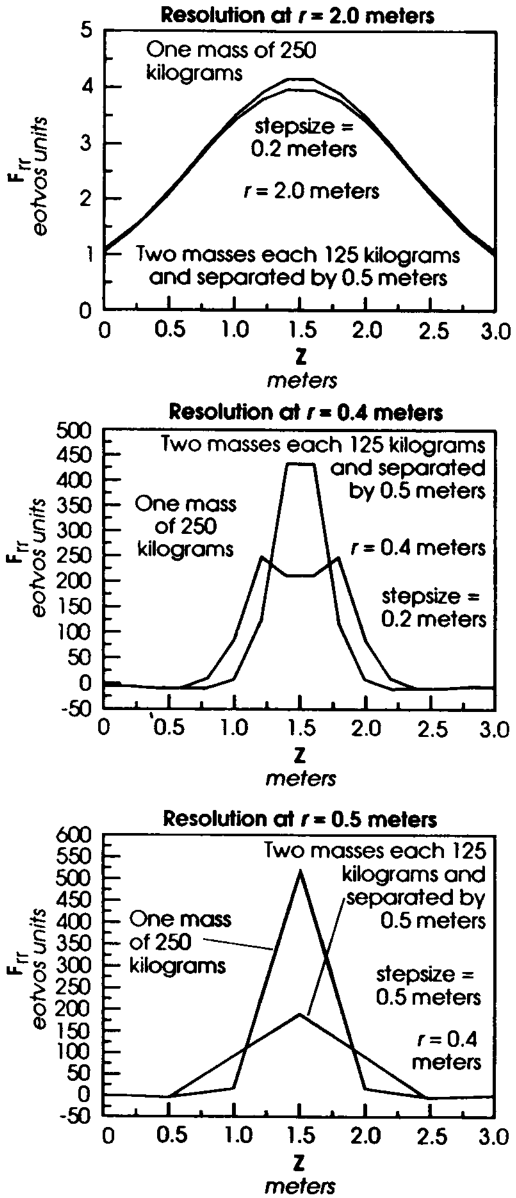


Figure 5: A comparison between the radial gradient of the gravitational field F_{rr} produced by a point mass of 250 kilograms and two point masses, each of 125 kilograms, separated by 0.5 meters. The simulated measurements were taken at intervals of 0.2 meters along the missile axis in figures 5a and 5b and 0.5 meters in figure 5c; and at a distance from the axis of 2.0 meters in 5a, and 0.4 meters in 5b and 5c.

Also, a party trying to spoof the gradiometer through the redistribution of mass must consider the aerodynamic consequences of such a strategy. For example, ideally a cruise missile should be designed such that during cruise the center of lift coincides with the lift position of just the wings—there is no drag acting on the horizontal tail surfaces.

CONCLUSION

Further study based upon more detailed information is required before a definite conclusion can be reached on the practical utility of using the gravity gradiometer for distinguishing conventional from nuclear cruise missiles. However, the present analysis is rather encouraging. More generally, the gravity gradiometer should be given serious consideration as a new and effective tool in future arms control agreements.

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APPENDIX 1

Gradients of the Gravitational Field

The geometry for computing the gravitational field produced by individual components of a cruise missile, which are approximated by lines of mass, is depicted in figure 6. The origin of the coordinate system O is chosen to coincide with the base plane of the hemispherical nose of a cruise missile. The coordinate system O' coincides with the point of measurement P and is located at a distance Z down the cruise-missile axis from the datum plane. A typical cruise missile component is approximated by a linear mass density of length L as shown. For a line of mass of length L the axial and radial components of the gravitational field are respectively given by

$$f_z = -G\sigma\left(\frac{1}{h_2} - \frac{1}{h_1}\right) \tag{1.1}$$

and

$$f_r = \frac{-G\sigma}{r}\left(\frac{(L + S - Z)}{h_2} - \frac{(S - Z)}{h_1}\right) \tag{1.2}$$

where r is the radial distance from the missile axis to the point of measurement P; S is indicated in figure 6; the mass per unit length σ is given by

$$\sigma = \rho A \tag{1.3}$$

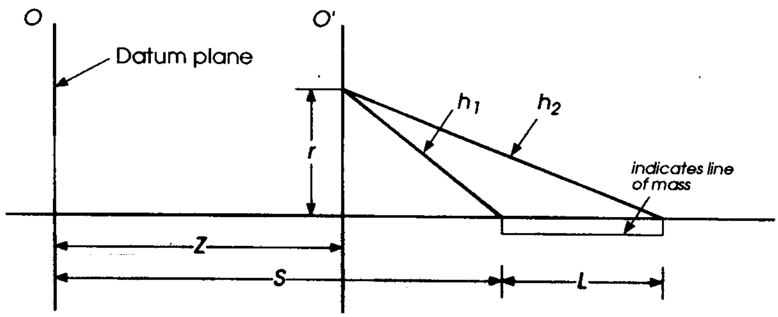

Figure 6: A depiction of the geometry used to compute the gravitational field produced by components within the longitudinal sections of a cruise missile. The coordinate system O coincides with the base plane of the hemispherical nose of a cruise missile. The coordinate system O' coincides with the point of observation P. A given component is approximated by a linear mass density of length L as shown. The remaining symbols are defined in appendix 1.

where ρ is the volume mass density of a component, with A the cross sectional area of a section in figures 1 and 2, and

$$\begin{aligned} h_2 &= \sqrt{r^2 + (L + S - Z)^2} \\ h_1 &= \sqrt{r^2 + (S - Z)^2} \end{aligned} \tag{1.4}$$

The mass associated with the skin of the hemispherical nose and the mass within the nose are to be approximated by point masses located at their respective centers of mass. For a point mass of mass m the axial and radial components of the gravitational field are given by

$$P_z = \frac{-Gm}{R^3}(Z - Z_{cm}) \tag{1.5}$$

$$P_r = \frac{-Gmr}{R^3} \tag{1.6}$$

where Z_{cm} is the location of the center of mass measured with respect to the coordinate system O and

$$R = \sqrt{r^2 + (Z - Z_{cm})^2} \tag{1.7}$$

The corresponding gradients are straightforward to obtain from equations 1.1–1.7. Because of azimuthal symmetry three gradients are nonzero. However only two of them are independent. For the line masses they are given by

$$f_{rr} = \frac{\partial f_r}{\partial r} \tag{1.8}$$

and

$$f_{rz} = \frac{\partial f_r}{\partial z} \tag{1.9}$$

In reality, it is unlikely that azimuthal symmetry will hold, so there would be five independent gradients to differentiate between conventional and nuclear cruise missiles. The corresponding total gradients are obtained by summing over the point and line masses

$$F_{rr} = \sum_i P_{rri} + \sum_i f_{rri} \tag{1.10}$$

and

$$F_{rz} = \sum_i P_{rzi} + \sum_i f_{rzi} \tag{1.11}$$

Because the gravitational field is divergenceless outside the mass distribution, the gradient F_{zz} for r outside the mass distribution is related to F_r and F_{rr} by

$$F_{zz} = -\left(F_{rr} + \frac{F_r}{r}\right) \tag{1.12}$$

where F_r is the total radial component of the gravitational field. Equations 1.10 and 1.11 were used in the calculations of the curves shown in figures 3 and 4.

NOTES AND REFERENCES

1. The prime motivation for developing this device was to improve inertial navigation of aircraft. For example, in the event of a nuclear war it is reasonable to assume that all electronic and visual navigation would be useless. So how could US bombers precisely navigate to designated coordinates? The vertical coordinate can be determined by a barometric altimeter and horizontal direction by stable and accurate gyroscopes. However with respect to a vertical direction of reference there are deviations in the vertical direction of the earth's gravitational field along the aircraft flight path. This produces a horizontal component of acceleration which results in an uncertainty in velocity. By using a specific system of gradiometers combined with accelerometers it is possible to accurately measure this horizontal component of the gravitational field in realtime and perform realtime course and trajectory prediction with the aid of onboard computers.

2. Mark A. Gerber, "Gravity Gradiometry: something new in inertial navigation", *Astronautics and Aeronautics* **16**, 5, May 1978, p.18; Milton B. Trageser, "Feasibility Model Gravity Gradiometer Test Results", presented at the AIAA Guidance and Control Conference, Boston, Massachusetts, 21 August 1975; Milton B. Trageser, "Floated Gravity Gradiometer Status Report," *Proceedings of the Second International Symposium on Inertial Technology for Surveying and Geodesy* (Banff, Canada, June 1981), p.537.

3. Private communication, Milton Trageser, Charles Stark Draper Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, March 1989.

4. Trageser, "Floating Gravity Gradiometer."

5. Private communication, Milton Trageser.

6. See, for example, Valerie Thomas, "Verification of Limits on Long-range Nuclear SLCMs," *Science & Global Security* 1, 1–2, appendix 1, p.41.

7. The average densities of fuel in sections 2 and 4 of figures 1 and 2 require some explanation. The average density of fuel in section 4 is notably less than the fuel in section 2. This has to do with an assumption regarding the inclusion of the mass associated with the retractable wings contained in section 4. By judicious choice the mass of the wings has been included in the airframe surrounding section 4. The mass density of the wings has been assumed to be equal to aluminum. The remaining volume in section 4 is occupied by the fuel. The average density of fuel is simply the mass of fuel in this section divided by the volume. This turns out to be less than the estimate for section 2. How does this affect the gradiometer scan? It is likely that the mass distribution from section 4 through section 6 for the conventional and nuclear versions is identical (See John R. Harvey, "Verification of Potential SLCM Limits," Lawrence Livermore National Laboratory preprint, June 1988). This is reflected in the simulated gradiometer scans of figures 3 and 4. In reality the actual scans of the conventional and nuclear versions would also be identical for sections 4–6, but they would probably differ in detail from the simulated scans. The main point is that the gradiometer scans of the conventional and nuclear versions would still be significantly different in sections 1–3.

8. Kosta Tsipis, "Cruise Missiles", *Scientific American* **26**, 2, February 1977, p.21.

9. These calculations neglect the transverse dimensions of the missile and its components. This is a good approximation at measurement distances that are relatively large compared to the transverse dimension. For the distances considered below it should be reasonable for estimating purposes. However, it is important to note that some potentially useful and measurable information may be lost in making this approximation.