

GUI_Missile_Flyout: A General Program for Simulating Ballistic Missiles

Geoffrey Forden

Massachusetts Institute of Technology (MIT), Program on Science, Technology, and Society

GUI_Missile_Flyout is a stand-alone program running under Windows for simulating ballistic missiles with 1, 2, or 3 stages in a framework with a round, rotating Earth. Users can easily input all the necessary parameters in an intuitive graphical user interface (GUI). A modest number of quantities can be interactively plotted on the Interface but the complete trajectory can be saved as either an Excel or Matlab file for further analysis. The trajectory can also be directly displayed in Google Earth for visualization. The GUI can be used to optimize pitch-over parameters to maximize range or aim at a specific target (entered, as is the launch site) through latitude–longitude pairs. In addition to an introduction to using the program, this article describes the integration of the three-degrees-of-freedom equations of motion and approximations made to the aerodynamic (such as a parameterized drag coefficient, C_d). The program is freely available from the website: <http://mit.edu/stgs/downloads>. Although written using MATLAB, it is not necessary for the user to own that program.

INTRODUCTION

Many contributions to the debate over national security policy have been made by analysts who have modeled ballistic missile trajectories.¹ With the continuing debates about North Korea's missile program, National Missile Defense, and the weaponization of space, it is likely that this will continue into the future. However, the simulation of ballistic missiles is not such a well known or available asset as might be hoped. In order to make such debates more accessible and to increase the exchange of knowledge, the author has written a program that can simulate ballistic missiles from short range artillery rockets to intercontinental ballistic missiles. Emphasis has been placed on making this program easy to use through a graphical user interface. It is provided to the community free of charge through the website <http://mit.edu/stgs/downloads>.

Received 12 December 2006; accepted 13 April 2007.
Address correspondence to Geoffrey Forden, MIT, E51-163, 77 Massachusetts Ave., Cambridge, MA 02139-4307. E-mail: Forden@mit.edu

In order for the user to understand the physical simulation, this article describes the equations of motion that are used to model the trajectory and their implementation in the program. Practical details, such as the methods used for gravity turns, are also discussed. It then goes on to illustrate the use of the program by analyzing three examples: an Iraqi Al Hussein with separating warhead, the differences in range caused by firing a Taepodong I missile either East or West, and the range of a Supergun with a Martlet 2G-1 projectile—the orbital launcher concept developed by Gerald Bull.² These examples should help the user model a wide range of missiles: from FROGs to space launch vehicles using the same program.

EQUATIONS OF MOTION

The motion of a missile moving through the atmosphere with a round, rotating Earth is given by³:

$$\vec{F} = m(t)\vec{g} - 2m(t)(\vec{\omega}_{\oplus} \times \vec{v}) + m(t)\vec{\omega}_{\oplus} \times \vec{r}_m \times \vec{\omega}_{\oplus} + \vec{T} + \vec{F}_{\text{aero}}(\vec{v}) \quad (1)$$

All the terms in this equation are functions of time, except the rotational velocity of Earth, $\vec{\omega}_{\oplus}$, which is expressed in radians per second. Even the local acceleration of gravity, \vec{g} , changes direction over the course of the trajectory (the missile's position relative to the center of the Earth is given by \vec{r}_m) and to a small degree, its magnitude. The mass, $m(t)$, changes as fuel and oxidizer is consumed. Thrust, \vec{T} , not only changes between stages and goes to zero as the last stage burns out, but also changes direction as the missile undergoes its "gravity turn."⁴ In fact, it is the changing direction of the thrust in the pitch-over that initiates the gravity turn. The aerodynamic forces, $\vec{F}_{\text{aero}}(\vec{v})$, depend on a number of variables, not just velocity, \vec{v} . A more detailed simulation than the one reported here would include lift forces, \vec{F}_L , as shown in Equation 2.

$$\vec{F}_{\text{aero}}(\vec{v}) = \vec{F}_D(\vec{v}) + \vec{F}_L(\vec{v}) \quad (2)$$

However, in order to have a more realistic simulation of lift forces, the attitude of missile with respect to its velocity, must be taken into account. That would introduce three more degrees of freedom (to make it a six-degree-of-freedom calculation) and considerably more parameters.⁵ This is also true for drag forces but the authors make the approximation that the angle of attack is small enough to ignore those complications. In general, ballistic missiles are considered too fragile to have very large angles of attack and, while even small angles of attack can have large effects, the authors make the approximation that only drag forces are significant. The further approximation is made that the drag force is determined by the missile's velocity, its cross-sectional area, A , a drag coefficient determined by the shape of the missile, C_d , and the atmospheric density, ρ , and that it opposes the missile's velocity as in Equation 3.

The absence of a dependency on the angle of attack is the essence of a three-degree-of-freedom calculation.

$$\vec{F}_D(\vec{v}) \cong -\frac{1}{2}\rho C_d A |v| \vec{v} \tag{3}$$

In calculating the drag force, the density of atmosphere is assumed to vary exponentially with height. For a rocket shape, drag coefficient, C_d , has its greatest variation as the missile passes the speed of sound, Mach 1. In general, the shape of the drag coefficient as a function of the missile's speed will depend on physical shape of the missile. However, as an approximation, it is assumed that the drag coefficient can be represented for all missiles simulated by a piecewise linear function as illustrated in Figure 1.⁶

Integrating the Equations of Motion

It has been common to approximate the equations of motion in Equation 1 by a flat, non-rotating Earth even if the trajectory is plotted on a round Earth. In that approximation, the forces acting on the missile can be approximated by Equation 4, where \hat{k} is a unit vector along the y axis.

$$\vec{F} \cong -mg\hat{k} + \vec{T} - \frac{1}{2}\rho C_d A |v| \vec{v} \tag{4}$$

In that approximation and starting with a zero initial velocity, a single plane contains all the forces acting on the missile as well as the complete trajectory. Although this is a reasonable approximation for tactical missiles such as the

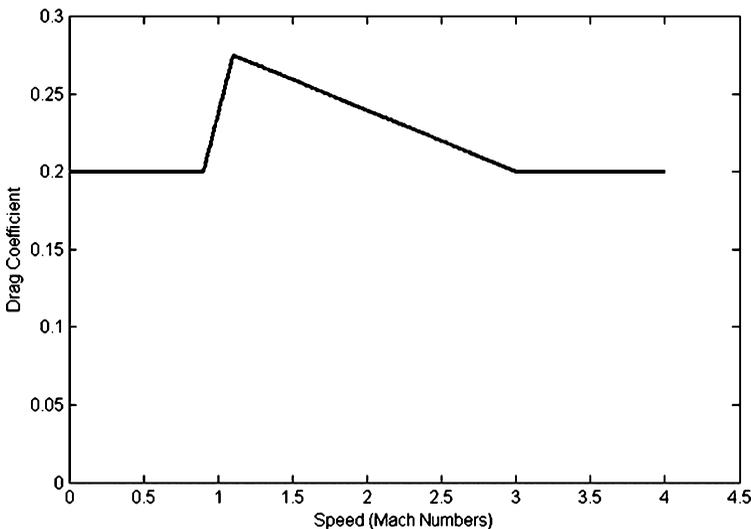


Figure 1: Approximation used for the drag coefficient as a function of missile speed.

300 km range SCUD-B, it is much worse for intercontinental ballistic missiles (ICBMs), as will be discussed in more detail in the section on Examples of Use.

However, modern computation software programs that specialize in matrix manipulation, such as MATLAB, make it possible to easily perform what until recently would have been considered very complex calculations for a personal computer. Thus, the full equation of motion, reproduced in Equation 5 with the drag approximation explicitly written, can be easily integrated using Simpson's rule.

$$\vec{F} = m(t)\vec{g} - 2m(t)(\vec{\omega}_{\oplus} \times \vec{v}) + m(t)\vec{\omega}_{\oplus} \times \vec{v} \times \vec{\omega}_{\oplus} + \vec{T} - \frac{1}{2}\rho C_d A |\vec{v}| \vec{v} \quad (5)$$

If the current position of the missile, its velocity, and thrust are represented by the vectors

$$\vec{r}_m = \begin{pmatrix} x_m \\ y_m \\ z_m \end{pmatrix}; \vec{v}_m = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}; \quad \text{and } \vec{T} = \begin{pmatrix} T_x \\ T_y \\ T_z \end{pmatrix},$$

whereas the Earth's rotational velocity and local acceleration of gravity are represented by

$$\vec{\omega}_{\oplus} = \begin{pmatrix} 0 \\ 0 \\ \omega_{\oplus} \end{pmatrix} \quad \text{and } \vec{g} = \begin{pmatrix} -MGx_m/r_m^3 \\ -MGy_m/r_m^3 \\ -MGz_m/r_m^3 \end{pmatrix}$$

respectively, where G is the universal gravitational constant and M is the mass of the Earth.

The MATLAB formulation of the equations of motion then becomes

$$\begin{aligned} m(t)\vec{a} = & m(t)\vec{g} - 2m(t)\text{cross}(\vec{\omega}_{\oplus}, \vec{v}) + m(t)\text{cross}(\text{cross}(\vec{\omega}_{\oplus}, \vec{r}_m), \vec{\omega}_{\oplus}) \\ & + \vec{T} - \frac{1}{2}\rho(r_m - R_E)C_d A \vec{v} \cdot \text{norm}(\vec{v}) \end{aligned} \quad (6)$$

where R_E is the Earth's radius, $\text{cross}(\vec{\omega}_{\oplus}, \vec{v})$ is MATLAB's routine for calculating vector cross products, and $\text{norm}(\vec{v})$ just returns the magnitude of the velocity vector. With this formulation, the velocity at the $n + 1$ step can be found from the velocity and position at the n th step by

$$\Delta \vec{v}_{n+1} = \left[\vec{g} - 2\text{cross}(\vec{\omega}_{\oplus}, \vec{v}_n) + \vec{T}_n - \frac{1}{2m(t_n)}\rho(r_{m,n} - R_{E,n})C_{d,n}A\vec{v}_n \cdot \text{norm}(\vec{v}_n) \right] \Delta t \quad (7)$$

and

$$\vec{v}_{n+1} = \vec{v}_n + \Delta \vec{v}_{n+1}$$

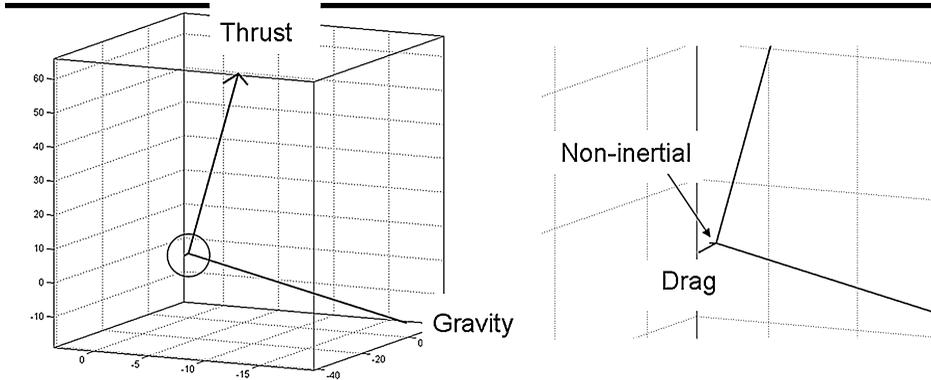


Figure 2: Forces (divided by missile mass) on a Taepodong I missile twenty-one seconds after launch in ft/s^2 . The right side shows an enlargement of the circled region to get a better view of the drag and non-inertial forces. Note also that the aspect ratio of the figure is such that the scales for x , y , and z directions are not the same, causing the thrust and gravity vectors to appear as if gravity was exerting a greater force than the thrust, which is not the case.

Likewise, the position at time $n + 1$ is given by

$$\vec{r}_{m,n+1} = \vec{r}_{m,n} + \vec{v}_{n+1} \Delta t$$

The relative magnitudes of the thrust, gravity, drag, and non-inertial forces are illustrated in Figure 2, which shows the different forces (normalized by dividing by the mass of the missile) on a two-stage Taepodong-I twenty-one seconds after lift off.

As can be seen, the thrust is considerably stronger than the Coriolis and centrifugal forces, which explains why simulations of tactical missiles can justifiably ignore these non-inertial effects. Also noticeable is the angle between the thrust and the drag vectors. Part of this is the angle chosen to view the vectors, which can enhance this impression. However, part of it is real. Because the drag vector is opposite the missile’s velocity, this implies that the angle between thrust and the missile velocity is fairly large for this particular simulation, which uses parameters optimized to maximize range.

In general, guided missiles liftoff vertically, that is, with a 90° attitude to the horizontal, and then pitch over into a “gravity turn” toward their target by slightly diverting the thrust off axis. In the missile this introduces a torque that eventually turns the missile in the direction desired. For a three-degree-of-freedom simulation, however, this torque cannot be modeled in a physically meaningful fashion. Instead, this program (as well as others) simply directs the thrust off vertical; causing the other components of the velocity to increase.⁷ In practice, this is accomplished using two parameters: the so-called loft angle (θ_{loft}) and the loft angle rate ($\dot{\theta}_{\text{loft}}$). The loft angle specifies the maximum angle off vertical the thrust vector can attain whereas the loft angle rate describes

how fast the thrust vector is diverted off vertical. Thus the direction of thrust to the local vertical is determined by

$$\theta = \begin{cases} 90^\circ - \dot{\theta}_{\text{loft}} t & 90^\circ - \dot{\theta}_{\text{loft}} t \geq \theta_{\text{loft}} \\ \theta_{\text{loft}} & \text{otherwise} \end{cases} \quad (8)$$

This formulation can introduce some less than physical behavior into the simulation. For instance, it appears that for a missile with a relatively low acceleration—and hence a relatively slow missile at low altitudes, such as the Taepodong I—the range maximization routine tends to push the thrust over faster than a real missile would be pushed over because it does not worry about torques breaking the missile body. Thus, if the drag force (which depends on v^2) is relatively small at lower altitudes, the missile can afford to use its thrust to increase the horizontal component of its velocity. For a faster missile, such as the SCUD-B, it appears better to rise higher and leave more of the atmosphere before there is much turn over. A future version of the software will attempt to solve this problem by introducing user-set limits to the optimization of the loft angle range.

IMPLEMENTING THE SIMULATION: GUI_MISSILE_FLYOUT

The program utilizes a graphical user interface (GUI) to change the missile parameters. This GUI is shown in Figure 3. These parameters are clustered into a number of functional groups. For instance, the parameters needed to characterize each stage are grouped together. Thus, the first stage’s dry and fueled masses are in the same panel as that stage’s burn time and specific impulse. The launch site’s parameters, its latitude and longitude as well as the azimuth of the missile’s trajectory and its loft angle and loft angle rate are grouped in a different panel.

As part of the program, four sets of missile parameters are included as prepackaged models. These are shown in Table 1. Additional missiles can be created by the user by changing the parameters shown on the GUI or creating a new missile from scratch by selecting the “Create New Missile from Panel” from the “Missile” pull-down menu on the tool bar. These new missiles can be saved, again from the “Missile” pull-down menu, for later use.⁸

A limited number of plots of trajectory quantities, such as range, altitude, and acceleration, are available to be plotted in the GUI. It is also possible to save a larger set of trajectory characteristics to either a MATLAB or EXCEL file by using the “Save Current Trajectory” option of the “Missile” pull-down menu on the tool bar. In addition, it is possible to save the trajectory to a GOOGLE EARTH readable file for visualization of the trajectory.

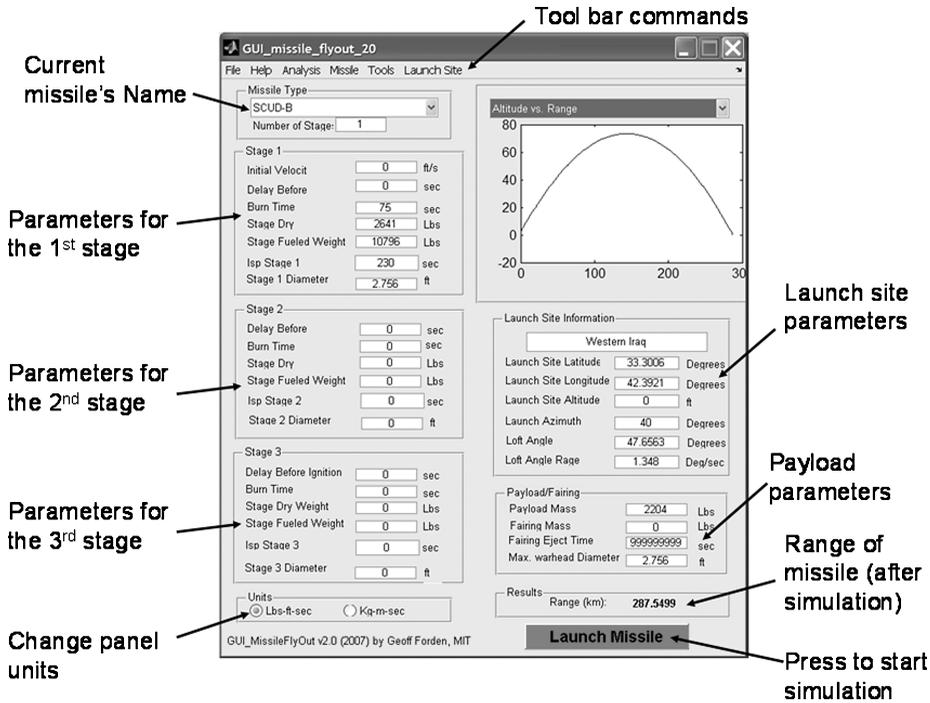


Figure 3: The Graphical User Interface used to change missile parameters.

EXAMPLES OF USE

The use of the program is illustrated in this section by three examples. These examples will point out some of the idiosyncrasies of the program. First, the effects on range of when a warhead is separated are examined. Second, the

Table 1: Missile parameters for packaged missile simulations.

	Taepodong I ¹				
	SCUD-B ²	Al Hussein	Nodong ³	Stage 1	Stage 2
Dry Weight (kg)	1198	1334	3900	3800	1198
Fueled Weight (kg)	4897	6073	19900	19900	4897
Specific Impulse (s)	230	230	240	240	230
Burn Time (s)	75	90	70	70	75
Diameter (m)	0.84	0.84	0.88	0.88	0.84
Payload (kg)	1000	191	1000	454	

¹Federation of American Scientists, <http://www.fas.org/nuke/guide/dprk/missile/td-1.htm>, accessed 5 December 2006.

²Parameters for both the SCUD-B and the Al Hussein: George N. Lewis and Theodore A. Postol, 1993, "Video Evidence on the Effectiveness of Patriot during the 1991 Gulf War." *Science and Global Security*, 4 (1993): 1–63.

³David C. Wright and Timur Kadyshev, "An Analysis of the North Korean Nodong Missile." *Science and Global Security*, 4 (1994): 129–160.

range differences between similar missiles launched Eastward and Westward are examined. And third, the ranges associated with the Iraqi Supergun are examined.

Al Hussein with a Separating Warhead

Iraq was aware of the problems associated with the breakup of the extended body⁹ of the Al Hussein and, in 1990, they decided that they needed a separating warhead to improve accuracy.¹⁰ The increased accuracy would come about because the extended missile body, while still attached to the warhead, tended to come down at large and fluctuating angles of attack. The resulting aerodynamic forces, including, most importantly lift forces that caused lateral accelerations many times that of gravity, resulted in wide dispersions in the impact point. A separated warhead would be more stable during descent and align better with the velocity vector. However, a separating warhead would actually decrease the theoretical ballistic coefficient, if naively calculated with the mass of the entire missile divided by the cross-sectional area.¹¹

An interesting technical question is then: What is the cost in terms of range of the missile by separating the warhead? This can be easily calculated using GUI_Missile_Flyout.¹² One way of doing this is to select Al Hussein in the missile type field and change the number of stages to two. Then the second stage's burn time should be changed to some large number (the program "drops" the previous stage's mass after the next stage starts), such as 5,000 s. Then set both the dry weight and the "fueled" weight to the payload weight, which will have the effect of producing a zero thrust stage—and zeroing out the payload weight to avoid double counting the payload. Warhead separation can be timed by changing the second stage's "Delay Before Ignition." Zero delay before second stage "ignition" corresponds to warhead separating immediately after the first stage burns out. For the Al Hussein, this corresponds to an altitude of 47 km and a speed of 2.3 km/s, as can be determined by running the program and saving the trajectory to an excel file. Forty-seven kilometers is very high but there is still appreciable atmospheric drag.

The simulation calculates a warhead's atmospheric drag by using the same drag coefficient, C_d , parameterization as is used for the entire missile and, indeed, the same C_d as is used for all missiles. Nevertheless, by dropping the first stage missile mass, the effective "acceleration"¹³ caused by drag is increased enormously, as illustrated in Figure 4, which shows the aerodynamic accelerations on the warhead with and without warhead separation.

Table 2 shows the ranges for the same Al Hussein with no separation, with separation well above the atmosphere (at 150 s after launch, which corresponds to 119 km altitude) and one with separation immediately after burnout. There is a net loss of range of 14 km if the warhead separates immediately after burnout—*assuming that somehow the Al Hussein without warhead separation*

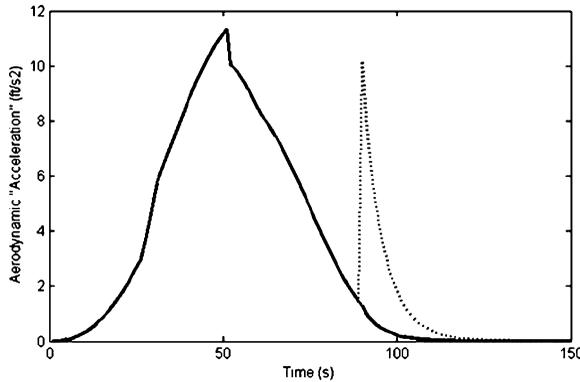


Figure 4: "Acceleration" on Al Hussein due to aerodynamic.

is stable and pointed along the velocity vector for the entire trajectory. Separation of the warhead even above the atmosphere (which is arbitrarily chosen to be 100 km) still causes a significant, 6 km decrease in range. Separating the warhead even later would, of course, reduce this range loss.

Effect of Easterly vs. Westerly Trajectories

What is the effect of launching a missile either to the East or to the West? This effect could have important political and policy significance. For instance, the United Nations Security Council Resolution 687 allowed Iraq to develop missiles with ranges less than 150 km. The Missile Technology Control Regime restrictions on sales of missile technology come into effect for ranges greater than 300 km.¹⁴ Iraq might very well have been cognizant of this after the first Gulf War as it tested its missiles in a Westerly direction; a direction that minimizes the range. But how much effect did this have?¹⁵ Again, this is easy to calculate using the program.

The packaged models have been optimized for trajectories that have azimuths 40° East of North (due East is 90°, South is 180°, and West is 270°). To be fair, each trajectory must be re-optimized for the new East and West trajectories. To do that, select a missile type and then edit the launch azimuth by simply overwriting the azimuth shown on the GUI. Missile parameters can then be optimized by selecting "Maximize Range" from the "Analysis" pull-down menu on

Table 2: Range vs. warhead time of separation.

Time of warhead separation after burnout (s)	Range (km)
0	673
150 (above the atmosphere)	681
No separation	687

Table 3: Ranges for missiles launched in easterly and westerly directions.

Missile	Easterly trajectory range	Westerly trajectory range
SCUD-B	288 km	282 km
Al Hussein	693	670
Nodong	973	934
Taepodong I	2349	2174

the tool bar for each model and each azimuth. The resulting maximum ranges are shown in Table 3. These results are shown graphically in Figure 5. It is interesting to note that using the parameters optimized for maximum range of a Taepodong I missile launched in an Easterly direction results in a trajectory that is nearly maximized for a Westerly trajectory as well.

The Supergun

One surprise of Iraq's weapons of mass destruction program, "discovered" shortly before the first Iraq war, was Project Babylon, a large caliber cannon that was intended to fire shells well over 100 km.¹⁶ However, this project should have been well known to the West since its chief scientist, Dr. Gerald Bull, had developed the concepts under both Canadian and U.S. funding.¹⁷ Bull envisioned this project, at least publicly, as being capable of launching small satellites into Earth orbit. At the same time he was helping Iraq develop a supergun for military purposes, although it appears that the Iraqi weapon would fire solid slugs and not rockets capable of self-propulsion.¹⁸

The program can be easily used to investigate this supergun. Bull, in his 1988 book on superguns, describes several self-propelled projectiles that could

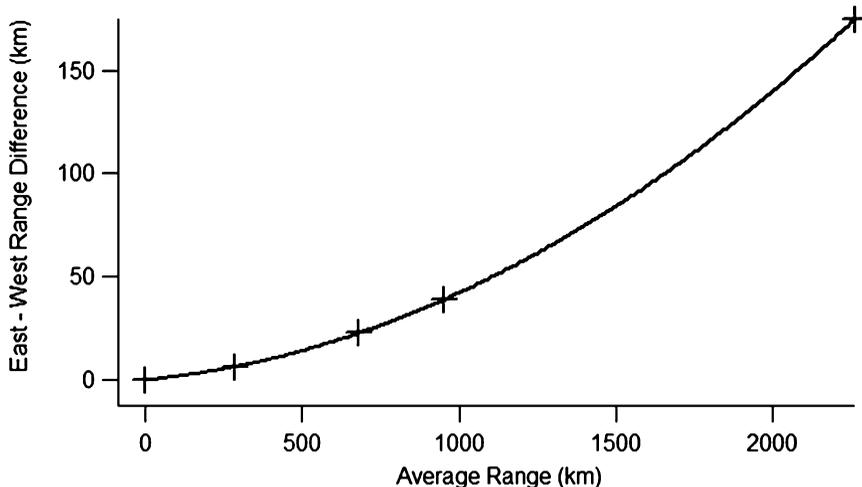


Figure 5: The range difference between missiles fired Easterly vs. Westerly. The curve is a quadratic fit to the generated data points, shown as crosses.

Table 4: Parameters for Supergun Martlet 2G-1⁴.

Stage	Stage dry weight	Weight of fueled stage	Specific impulse (I_{sp})	Burn time	Stage diameter
1	28.5 kg	132.9 kg	272 s	40 s	0.26 m
2	14.4	43.1	272	16	0.26
3	3.6	11.3	272	4	0.18
Payload	4.1				

⁴Bull and Murphy, p. 229.

be launched. Table 4 describes the operational parameters of one such projectile/rocket. As described, the supergun fires a projectile from a long gun of three hundred meters in length with a bore diameter of 1 meter. The projectile, named Martlet 2G-1, consists of a three-stage rocket with a maximum diameter of 0.26 meters (the projectile is enclosed in a sabot during its passage through the gun to seal in the gases generated by the cannon's charge). Bull calculated that the supergun would give the projectile a velocity of 1.8 km/s (or 6000 ft/s, the units Bull uses).

Assuming that is true, the supergun can be modeled by selecting "Create New Missile From Panel" in the "Missile" pull-down menu on the tool bar. Enter a name for the model (such as "Martlet 2G-1") and set the number of stages to 3. Then fill in the three panels for the stages and the panel for the payload weight with parameters as given in Table 4. When the initial velocity, in the first stage panel, is changed a dialog pops up that asks for the elevation of the initial velocity. Set this to 55 degrees; this is the angle from horizontal. The dependence on this variable shall be examined later on. Note that the parameters can be inputted in either kg-m-s or lbs-ft-s.

Once an initial velocity has been inputted, the program turns off the gravity turn procedure. Once this has been turned off, the pitch-over algorithm cannot be reactivated for a model even by setting the initial velocity to zero; that specific model will always simulate an unguided missile even if it is renamed or copied. It should be noted that the program is not limited to superguns and the same procedure could be used to simulate other artillery rockets such as a Free Rocket Over Ground (or FROG) by just "resetting" the initial velocity from zero to zero, which causes the elevation dialog to pop up. The program can do this as long as the integration step is set a longer time period than the rocket would stay on its launch rail. Because this is currently fixed to one second, there is no problem simulating FROGs or other such artillery rockets.

The first step is to calculate the range of the supergun with a solid slug and not a self-propelled rocket being launched. This can easily be simulated by setting the "Delay Before Ignition" to some large number, such as 5000 s. With the azimuth set to 90° (or East), we get a range of 157 km without any self-propulsion.¹⁹ Turning the rocket motors on, but without the initial velocity, gives a maximum range of 2224 km, indicating the contribution of the rocket

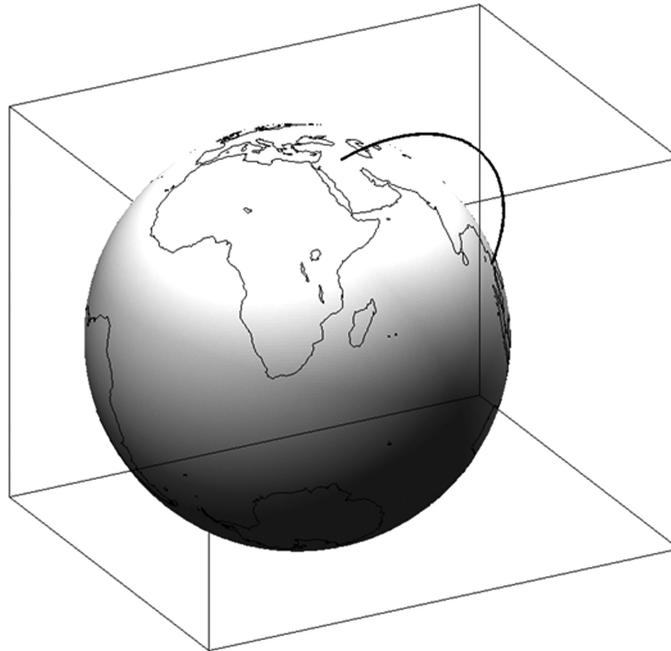


Figure 6: Trajectory of the Supergun (Martlet 2G-1).

propulsion. Now, the complete supergun (canon plus Martlet 2G-1 rocket) has a range of 5738 km, considerably larger than either previous trajectory. This is shown in Figure 6.

As noted earlier, the initial velocity for the supergun was set at 55° , which would be a considerably greater elevation than one might expect for a projectile in a vacuum (or the limiting loft angle of a missile in powered flight high above the Earth's surface). This can be examined by simply varying the initial

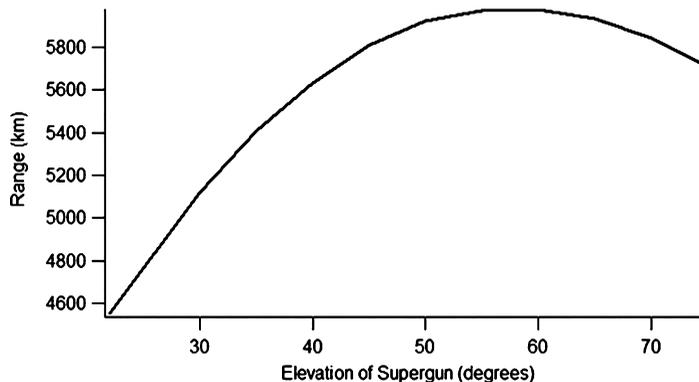


Figure 7: Range vs. elevation of supergun (with 1.8 km/s muzzle velocity).

velocity's elevation. The resulting plot is shown in Figure 7. It is interesting to note that the Paris gun, with a range of approximately 130 km, had an elevation of 50° .²⁰

NOTES AND REFERENCES

1. A recent example is David K. Barton *et al.*, *Report of the APS Study Group on Boost-Phase Intercept Systems for National Missile Defense*, American Physical Society, July 2003.
2. G. V. Bull and C. H. Murphy, *Paris Kanonen—The Paris Guns (Wilhelmgeschuetze) and Project HARP: The Application of Major Calibre Guns to Atmospheric and Space Research* (Herford, Germany: Verltag E.S. Mittler & Sohn GmbH, 1988).
3. See, for example, R. C. Callaway Jr., “6.2 Powered Flight Through Atmospheres,” *Handbook of Astronautical Engineering*, ed. Heinz Koelle (New York: McGraw-Hill Book Co., 1961), pp. 6-24–6-50.
4. The definition of a gravity turn has the thrust exactly opposite the velocity vector. For a missile launched vertically, there must, however, be a pitch-over program where the thrust is directed slightly off axis. Glen J. Culler and Burton D. Fried, “Universal Gravity Turn Trajectories,” *Journal of Applied Physics* 28(6) (June 1957): 672–676.
5. There are eleven additional parameters needed for modeling lateral dynamics alone. John H. Blakelock, *Automatic Control of Aircraft and Missiles*, Second Edition (New York: John Wiley & Sons, Inc., 1991), p. 117.
6. This is a piece-wise linear interpolation and approximation of a “typical” rocket’s drag vs. Mach number. No attempt has been made to “fine tune” this parameterization because it is an approximation used for all missile shapes simulated by the program. See George Sutton and Oscar Biblarz, *Rocket Propulsion Elements*, 7th edition (New York: John Wiley & Sons, Inc., 2001), p. 108.
7. Holding the thrust angle fixed relative to the horizon yields a very good approximation for finding the maximum range. Culler and Fried, *op. cit.*
8. Saving the trajectory is different from saving the missile parameters and does not result in saving any changes the user might have made in a given model.
9. See George N. Lewis and Theodore A. Postol, “Video Evidence on the Effectiveness of Patriot during the 1991 Gulf War,” *Science and Global Security* 4(1) (1993): 1–63.
10. UNMOVIC, Summary of the compendium of Iraq’s proscribed weapons programmes in the chemical, biological and missile areas. S/2006/420, June 2006.
11. The ballistic coefficient, or beta, is defined as the object’s mass times a coefficient dependent on the shape and divided by the cross-sectional area.
12. In reality, the Al Hussein with warhead still attached was unstable on reentry. This effect on its range is not modeled because the model is only a three-degrees-of-freedom simulation.
13. Acceleration cannot be broken down into component “accelerations” the way forces can. However, the relative importance of aerodynamic forces can be examined by dividing the component forces by the missile mass.
14. Complete rocket systems are controlled as Category I if they are capable of delivering at least 500 kg a distance of at least 300 km. The U.S. State Department,

<http://www.state.gov/t/ac/trt/5073.htm>, *Missile Technology Control Regime*, accessed December 12, 2006.

15. In practice, of course, Security Council action would only be triggered if the excess range was significantly above the threshold. For instance, Iraq's Al Samoud II was termed a "prima facie" case of violations because it exceeded the limit by some 35 km whereas the Al Fatah was much more ambiguous. Hans Blix, "Briefing of the Security Council, 27 January 2003: An update on inspections," available at <http://www.unmovic.org/>, accessed December 6, 2006.

16. Chris Cowley, *Guns, Lies and Spies: How We Armed Iraq* (London: Hamish Hamilton, 1992), p. 12.

17. See Bull and Murphy, *op. cit.*

18. Richard Butler, Report: Disarmament, Appendix I: Status of the Material Balances in the Missile Area, paragraphs 77–79, January 1999, S/1999/94.

19. Simulating the Paris Gun with a 106 kg shell, the program estimates a range of 111 km whereas the range is reported to be approximately 130 km. The difference might well be due to the aerodynamic lift, which is not simulated in this three-degree-of-freedom simulation. Bull and Murphy simulate the Paris gun and not only get a range of 130 km but also a large angle of attack for approximately the last 120 s of its flight. These angles of attacks, up to 30 degrees, would certainly produce large lift forces. Bull and Murphy, *op. cit.*, p. 128.

20. Bull and Murphy, *op. cit.*, p. 33.