Climate Change, Nuclear Power and Nuclear Proliferation: Magnitude Matters

On-Line Appendices

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General notes to Appendices:[1]

1. In all cases the designation "Mt", accepted for use with the S.I. system, denotes millions of metric tonnes. This is to be distinguished from "MT" which is often used in the U.S. literature to denote metric tonnes.
2. In all cases power production, *e.g.,* P_{LWR} and P_{FR}, is measured in GWe-yr/yr, to be interpreted as actual electric power production, as distinguished from the commonly quoted electrical power capacity. For example 111 power plants each with 1 Gigawatt-electric (GWe) capacity, operating at capacity factor 0.9, produce 99.9 GWe-yr/yr.
3. All flows assigned to year *n* are assumed to occur on January 1 of year *n*, and all stocks assigned to year *n* are assumed to be assayed at mid-year, July 1, of year *n*.

Appendix 1: Committed Energy Production

Ignoring startup effects, in a system of power plants that has been operating at a nearly steady power level for a period of time long compared to plant lifetimes, the average plant will be at the mid-point in its lifetime. Thus the amount of additional energy that is committed to be produced by the existing plants

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during their remaining lifetimes, after time t_0 when further construction ceases, is $E_{com} = P(t_0)\,\tau_{PP}/2$, where $P(t_0)$ represents the power level at time t_0 and τ_{PP} is the expected power plant lifetime.

This simple result can be generalized for a continuously exponentially growing or decaying system in which all further construction ceases at time t_0. The power production curve before time t_0 is given by

$$P(t < t_0) = P(t_0)e^{m(t-t_0)} \qquad \text{Eq. A1.1}$$

where $m > 0$ is the annual growth rate, and $m < 0$ the annual decay rate. New construction and decommissioning must have the same exponential time dependence, so we find

$$\frac{dP(t < t_0)}{dt} = mP(t_0)e^{m(t-t_0)} = C_0e^{mt} - D_0e^{mt} \qquad \text{Eq. A1.2}$$

Since each plant must be decommissioned τ_{PP} years after it was commissioned, we also have

$$D_0e^{mt} = C_0e^{m(t-\tau_{PP})} \qquad \text{Eq. A1.3}$$

from which we can find that the decommissioning rate is given by

$$D_0e^{mt} = \frac{mP(t_0)e^{m(t-t_0-\tau_{PP})}}{1-e^{-m\tau_{PP}}} \qquad \text{Eq. A1.4}$$

This formula is valid even beyond t_0, since plants constructed prior to that time still need to be decommissioned at the originally projected rate, until the last plant is decommissioned at $t = t_0 + \tau_{PP}$ and $P = 0$. Unlike simple exponential decay, the decommissioning rate is maximum at $t = t_0 + \tau_{PP}$, and P reaches zero. We can then integrate the decommissioning rate to find the power profile after new construction ends:

$$P(t > t_0) = P(t_0)\left[1 - \frac{e^{-m(t_0+\tau_{PP})}}{1-e^{-m\tau_{PP}}}\left(e^{mt} - e^{mt_0}\right)\right] \qquad \text{Eq. A1.5}$$

This can finally be integrated over the remaining committed energy production from time t_0 to time $t_0 + \tau_{PP}$ to give the total committed power production:

$$E_{com} = P(t_0)\tau_{PP}\left[\frac{m\tau_{PP} - \left(1-e^{-m\tau_{PP}}\right)}{m\tau_{PP}\left(1-e^{-m\tau_{PP}}\right)}\right] \qquad \text{Eq. A1.6}$$

For the estimates given in the main text τ_{PP} is taken to be 60 years, and m is chosen to fit the annual power production 30 years before the designated commitment point. Committed additional CO_2 emissions, mining and used fuel production for different types of power plants can be computed from E_{com}.

To check the effect of the approximations used in deriving equation A1.6, an explicit calculation was performed for the committed energy after 2100 in the scenario where light-water reactors (LWRs) provide all of the nuclear power of figure 3. Linear decline was assumed for currently existing plants between today and 2050, allowing specific decommissioning and construction dates to be defined as needed to fill the remainder of the power curve. The explicitly calculated committed power in 2100 agrees with equation A1.6 to 2 percent.

Appendix 2: Equations for Stocks and Flows of Uranium, Plutonium and Minor Actinides Associated with Light Water Reactors

A2.1 Transuranics in Existing Used Nuclear Fuel

The transuranics (TRU) in existing used nuclear fuel, denoted $TRU_{UNF}(0)$ in the following equations, is required as an initial condition in the time-dependent calculations of TRU in used nuclear fuel. It was estimated at 2580t of TRU on the basis of the IAEA "Overview of Global Spent Nuclear Fuel Storage,"[1] the "IAEA Nuclear Technology Review"[2] and the "Global Fissile Material Report 2009."[3]

A2.2 Uranium Fueling

The natural uranium consumed to produce 1 GWe-yr of nuclear electricity from LWRs, denoted U_c in the following equations, was evaluated at 204.7t, based on figure A-4.2 in the M.I.T. "Future of Nuclear Power" report[4] and associated calculations, assuming a relatively aggressive 0.25 percent Uranium-235 concentration in the enrichment tails to maximize uranium utilization, and 4.51 percent fuel enrichment. The M.I.T. report assumes 33 percent efficiency for the LWRs, which is adopted here. Adjustment was made for the assumed capacity factor of 0.9. The 204.7t of natural uranium required for 1 GWe-yr, enriched to 4.51 percent, with 0.25 percent tails, corresponds to 22.15t of initial heavy metal (iHM) fuel. 6.89 kg of Separative Work (SWU) is required for each kg of this fuel. Within this fuel is almost exactly 1t of Uranium-235, so the annual flow of Uranium-235 to LWRs, in tonnes, is almost exactly equal to P_{LWR} in GWe-yr/yr.

It was assumed that one year is required for processing, enrichment and fabrication between the time uranium is considered to be “mined” and when it is used as fuel. Since the residence time of fuel in LWRs is assumed to be 4.5 years, when a net new reactor is started in the calculations, 4.5 yearly loads of uranium are assumed to be required. For simplicity, if a reactor is decommissioned and another commissioned in the same year, it is assumed that only one year of fresh fuel is required for those reactors, that year.

A2.3 Transuranic Stock in Light Water Reactors

Time-averaging of the composition of LWR fuel[5] during burn from 0 to 50 MWd/kg provides an estimate of the inventory of TRU in a 1 GWe LWR. Taking into account a capacity factor of 0.9, this gives the TRU stock for 1 GWe-yr/yr of electricity production of

$$TRU_{LWRc} = 0.80\text{t} \qquad \text{Eq. A2.1}$$

The total stock of TRU in LWR cores is then simply $TRU_{LWRc}P_{LWR}(n)$.

A2.4 Transuranic Flow to Light Water Reactor Used Nuclear Fuel Stock

The flow of TRU from LWRs to the total stock of LWR used nuclear fuel per GWe-yr was estimated on the basis of the M.I.T. report, table A4-1, assuming burnup of 50 GWd per metric tonne of iHM, and adjusting for the assumed 0.9 capacity factor. The LWR production rate of TRU, denoted $TRU_{p,LWR}$ in the following equations, equals 0.3207t/yr, of which 0.295t is plutonium. When the count of reactors is reduced it is assumed that TRU_{LWRc} flows to the used nuclear fuel, but if a reactor is decommissioned in the same year that another is commissioned, then only $TRU_{p,LWR}$ of TRU is assumed to be produced from those reactors, that year, and to flow into the stock of LWR used nuclear fuel.

The proposed statutory capacity of Yucca Mountain is 70,000t of heavy metal, of which 1.447 percent or 1013t, is TRU.

A2.5 Evolution Equations for Stocks and Flows

The evolution equations are formulated as difference equations, with time-step of one year. As noted above, all flows(n) are considered to occur on January 1 of year n, and all stocks(n) are evaluated on July 1 of year n.

Taking into account the assumed one-year delay between mining and fueling, the change in the stock of mined uranium is given by

$$U_m(n) = U_m(n-1) + U_c P_{LWR}(n+1) + 3.5 U_c Max\left[P_{LWR}(n+1) - P_{LWR}(n), 0\right] \quad \text{Eq. A2.2}$$

where U_m (-1) is set at zero, so that $U_m(0)$, supplying the uranium for the first year of operation in the calculation is included. (The last term includes a factor of 3.5, rather than the full residence time of 4.5 years, because the previous term is evaluated at time n + 1.)

The evolution of the stock of TRU in LWR used nuclear fuel due to LWR operation is given by:

$$\begin{aligned} &TRU_{UNF}(n) = TRU_{UNF}(n-1) \\ &+TRU_{p,LWR} P_{LWR}(n-1) + TRU_{LWRc} Max\left[P_{LWR}(n-1) - P_{LWR}(n), 0\right] \end{aligned} \quad \text{Eq. A2.3}$$

Note that when fast reactors (FRs) are included in Appendix 3, they will add important terms to this equation.

Appendix 3: Equations for Stocks and Flows of Plutonium and Minor Actinides Associated with Fast Spectrum Fission Reactors

A3.1 Transuranic Fueling Flow to Fast Spectrum Fission Reactors

The burnup rate (BU_{FR}) and TRU mass fraction (f_{TRU}) for a modern TRU-burning fast reactor design has recently been calculated as a function of conversion ratio (CR). The annual fueling rate, in metric tonnes, required to produce 1 GWe-yr can be calculated from these as

$$L_{FR}(t) = \frac{1\ GWe}{\eta_{th}} \frac{365.25d}{BU_{FR}(GWthd/t)} f_{TRU} \quad \text{Eq. A3.1}$$

where d indicates days. The thermal efficiency, η_{th}, for these designs is estimated at 38 percent. Figures 2-20 and 2-21 in Bays *et al.*[6] provide BU_{FR} and f_{TRU} as functions of CR, but numerical values are not available. Figure A2-1 provides a fit to L_{FR} based on values read from these figures, used in the following calculations. Only CR values between 0.5 and 1.5 have been used in the calculations.

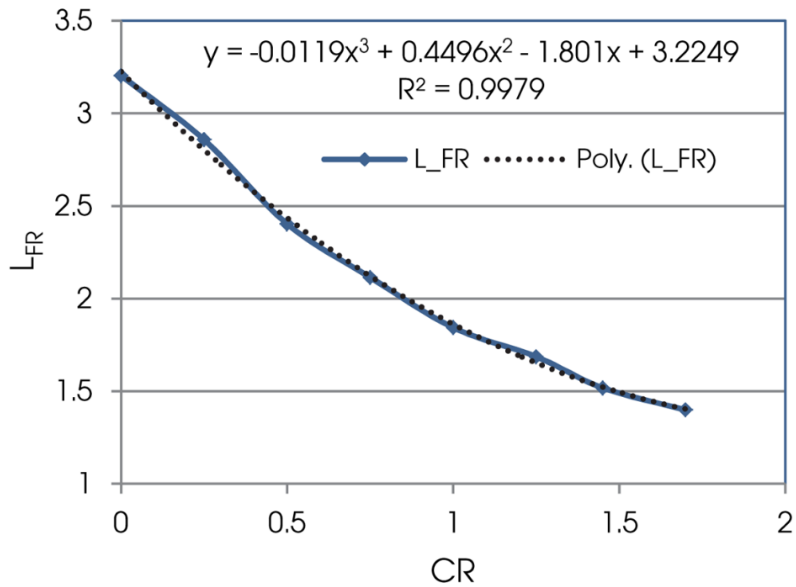

Figure A2-1. L_{FR}, the TRU load to fast reactors in order to produce 1 GWe-yr of electrical energy, from graphically reported calculations for burnup and fraction of TRU in fast reactor heavy metal, as a function of conversion ratio, *CR*.

The residence time of the fuel in the reactor is assumed to be τ_{FR} = 4 years, taking into account capacity factor, and consistent with estimates of damage tolerance by Hoffman *et al.*,[7] 2006. (Note that in these calculations τ_{FR} is constrained to integer values.) It is assumed that each additional GWe-yr/yr of installed FRs requires a load of $\tau_{R,FR}L_{FR}$.

A3.2 Transuranic Stock

The stock of TRU in a FR is a complex calculation, due to fuel shuffling, changes in reactivity, and other effects. Since $L_{FR} \sim 2B_{FR}$ and $|CR - 1| < 0.5$ in these calculations the effects of various approximations are in the few percent range. Here we take an approximation that has the benefit that it allows an accurate check of stocks against flows - *i.e.,* in all fast reactors at all times the rate of growth of TRU is $B_{FR}(CR\text{-}1)$/GWe-yr. This gives on July 1st of any year an in-reactor inventory of

$$\tau_{R,FR}L_{FR} + B_{FR}(CR-1)/2.$$

The total inventory of TRU in the fuel cycle depends on τ_F as discussed in the main text. We take τ_F to vary between 2 years, for on-site cooling, reprocessing and fuel fabrication, to 11 years for cooling, transportation to a centralized fuel recycling center, reprocessing, fuel fabrication, and return to the fast reactor as in Dixon *et al.*[8] TRU in LWR used nuclear fuel that is to be used to fuel FRs is given an effective τ_F of one year.

A3.3 Transuranic Unload Flow from Fast Reactors

At η_{th} = 38 percent, 1 GWe-yr of electrical energy requires 2.632 GWth-yr of thermal energy. Since the fission of 1t of heavy metal results in 1000 GWth-d of thermal energy, this means that B_{FR}, the burned heavy metal per GWe-yr is 0.9611t. The amount of TRU unloaded after this much energy production is

$$B_{FR}CR + (L_{FR} - B_{FR}) = L_{FR} + B_{FR}(CR-1) \qquad \text{Eq. A3.2}$$

It is assumed that when an FR is decommissioned, and another is commissioned, the net unload flow is not affected. The unload flow from a net decommissioned FR would be

$$\tau_{R,FR}L_{FR} + B_{FR}(CR-1) \qquad \text{Eq. A3.3}$$

since it is assumed that the decommissioning would occur at the end of a burn cycle. Note, however, that in the calculations of figures 5–8 there is never a net decrease of P_{FR}.

A3.4 Processing Losses

Many references assume one percent waste loss during reprocessing at the industrial scale. Consistent with these, in the calculations of this paper we take F_w = 0.01.

A3.5 Evolution Equations for Stocks and Flows

The total fueling flow needed for the FRs on Jan 1 of year n is given by

$$F_{FR}^{tot}(n) = L_{FR}P_{FR}(n) + \left(\tau_{R,FR} - 1\right)L_{FR}\left[P_{FR}(n) - P_{FR}(n-1)\right] \qquad \text{Eq. A3.4}$$

whereas the total fueling flow available from prior operation of FRs is given by

$$\begin{aligned} F_{FR}^{FR}(n) &= \left[L_{FR} + B_{FR}(CR-1)\right]P_{FR}(n-\tau_F-1)/\left(1+F_w\right) \\ &+\left(\tau_{R,FR}-1\right)L_{FR}Max\left[P_{FR}(n-\tau_F-1) - P_{FR}(n-\tau_F),0\right]/\left(1+F_w\right) \end{aligned} \qquad \text{Eq. A3.5}$$

including the source from net decommissioning of fast reactors. The fueling flow from the stock of LWR used nuclear fuel is just the difference between these.

To get to the full TRU evolution equation, the additional contribution to TRU_{UNF} from LWRs must be included, as must loss to waste. Furthermore, the TRU_{UNF} must be reprocessed and fabricated into fuel, requiring an assumed period of one year. Taking this in account, we have:

$$
\begin{aligned}
&TRU_{UNF}(n) = TRU_{UNF}(n-1) \\
&-(1+F_w)\left\{L_{FR}P_{FR}(n+1)+\left(\tau_{R,FR}-1\right)L_{FR}Max\left[P_{FR}(n+1)-P_{FR}(n),0\right]\right\} \\
&+\left[L_{FR}+B_{FR}(CR-1)\right]P_{FR}(n-\tau_F) \\
&+\left(\tau_{R,FR}-1\right)L_{FR}Max\left[P_{FR}(n-\tau_F)-P_{FR}(n-\tau_F+1),0\right] \\
&+TRU_{p,LWR}P_{LWR}(n-1)+TRU_{LWRc}Max\left[P_{LWR}(n-1)-P_{LWR}(n),0\right]
\end{aligned}
\qquad \text{Eq. A3.6}
$$

The total stock of TRU in the FR system at any time, n, is given by the sum of the TRU in the FRs, plus the total fueling for year $n+1$, multiplied by $(1+F_w)$, plus all $FR \rightarrow FR$ fueling in process for other years.

$$
\begin{aligned}
&TRU_{FRS}(n) = \left[\tau_{R,FR}L_{FR}+0.5(CR-1)B_{FR}\right]P_{FR}(n) \\
&+(1+F_w)L_{FR}\left\{P_{FR}(n+1)+\left(\tau_{R,FR}-1\right)\left\{Max\left[P_{FR}(n+1)-P_{FR}(n),0\right]\right\}\right\} \\
&+\left[L_{FR}+B_{FR}(CR-1)\right]\sum_{m=n+1-\tau_F}^{n-1}P_{FR}(m)
\end{aligned}
\qquad \text{Eq. A3.7}
$$

where we are not including the possibility of net reduction of FRs over time, since that does not occur in the calculations shown here, nor are we allowing for the case of overproduction from FRs, where the TRU unload from year $n - \tau_F - 1$ is greater than $(1+F_W)$ times the loading requirement for year n.

The flow of TRU to the waste stream is easily evaluated as F_w times the total flow to fueling FRs.

$$
\begin{aligned}
&TRU_W(n) = TRU_W(n-1) \\
&+F_w\left\{L_{FR}P_{FR}(n)+\left(\tau_{R,FR}-1\right)L_{FR}\left[P_{FR}(n)-P_{FR}(n-1)\right]\right\}
\end{aligned}
\qquad \text{Eq. A3.8}
$$

again assuming no net decommissioning of fast reactors.

It is helpful, to check the numerical implementation of these equations, to evaluate the changes in stocks from the start to the end of the calculation, against the summed flows. For example, for cases with monotonically rising P_{FR}

$$
\begin{aligned}
&TRU_{UNF}(N)-TRU_{UNF}(0) = TRU_{p,LWR}\sum_{n=0}^{N-1}P_{LWR}(n)+TRU_{LWRc}[P_{LWR}^{\max}(1,N)-P_{LWR}(N)] \\
&-(1+F_w)L_{FR}\left[\sum_{n=1}^{N}P_{FR}(n+1)+\left(\tau_{R,FR}-1\right)P_{FR}(N+1)\right]+[L_{FR}+B_{FR}(CR-1)]\sum_{1}^{N-\tau_F}P_{FR}(n)
\end{aligned}
$$

Eq. A3.9

We can also sum the flows into and out of the TRU pool associated with the FR system, giving the result:

$$TRU_{FRS}(N) = \left(1+F_w\right)L_{FR}\left\{\sum_{n=1}^{N} P_{FR}\left(n+1\right)+\left(\tau_{R,FR}-1\right)P_{FR}(N+1)\right\} - [L_{FR}+B_{FR}(CR-1)]\sum_{n=1}^{N-\tau_F} P_{FR}\left(n\right)$$
$$+B_{FR}\left(CR-1\right)\left[\sum_{n=1}^{N-1} P_{FR}\left(n\right)+0.5P_{FR}\left(N\right)\right] - F_w L_{FR}\left[\sum_{n=1}^{N} P_{FR}(n)+\left(\tau_{R,FR}-1\right)P_{FR}(N)\right] \quad \text{Eq. A3.10}$$

These and other checks have been implemented in the calculations here. The results are accurate to numerical precision.

A3.6 Fraction of Fast Reactors in a "Balanced" Steady State System

From these equations is it straightforward to evaluate the fraction, f_{FR}, of total nuclear electric power in fast reactors with $CR < 1$ that will burn (and dispose as waste) exactly the TRU that is produced from a fraction ($1 - f_{FR}$) of total nuclear electric power in thermal reactors, in a steady-state situation. This amounts to solving the evolution equation for TRU_{UNF} (Eq. A3.6) for a situation in which all terms are independent of *n*.

$$f_{FR} = \frac{TRU_{p,LWR}}{TRU_{p,LWR}+B_{FR}\left(1-CR\right)+F_w L_{FR}} \quad \text{Eq. A3.11}$$

For $CR = 0.5$, $L_{FR} = 2.43$t, and $f_{FR} = 0.388$.

A3.7 Growth and Decay Rates of Fast Reactors with Zero Transuranic Input

One can use the above equations to consider growing or decaying situations with net zero input of TRU. This again amounts to solving equation 3.6, but now making only TRU_{UNF} independent of *n* (and neglecting any source from LWRs). For the growth case, one arrives simply at

$$CR_{m>0} = 1+\frac{\left(1+F_W\right)(1+m\tau_{R,FR})\left(1+m\right)^{\tau_F}-1}{\left(B_{FR}/L_{FR}\right)} \quad \text{Eq. A3.12}$$

where $m > 0$ is the annual growth rate of FRs. For physical intuition, it is helpful to look at the limit of small *m*, which gives

$$B_{FR}\left(CR_{m>0}-1\right) = L_{FR}\left[F_w + m\left(\tau_{R,FR}+\tau_F\right)\left(1+F_w\right)\right]. \quad \text{Eq. A3.13}$$

The left-hand side is the amount of extra TRU produced per year per GWe-yr, while the right-hand side represents the needs for the next year in terms of sustaining the current FRs against loss to waste and the needed growth in stock of the reactors and the fuel reservoir, taking into account loss to waste.

Equation A3-12 has been tested against the time-dependent numerical calculation. Setting LWR power to zero and using A3-12 for the relation between *CR* and *m*, there is no change in LWR used nuclear fuel, to numerical precision.

Equation A3-12 however is not in good agreement with equation 2 of Piet *et al.*[9]:

$$CR_{m>0} = e^{m(\tau_F + \tau_{R,FR})}\left[1 + m\left(\tau_{R,FR} - 1\right)\right]$$

even when setting $F_w = 0$. The derivation of this equation is not given. It is notable that the ratio B_{FR}/L_{FR} does not appear. The authors evaluate two cases with $m = 0.0175$. For $\tau_{R,FR} = 4$, $\tau_F = 2$, the "example for onsite recycling", $CR = 1.17$ is required per their equation 2, cited above, and for the case of $\tau_{R,FR} = 4$, $\tau_F = 11$, the "example for offsite recycling", $CR = 1.37$ is required.

Table A3 - I compares the results of the two equations in the limit $F_w = 0$.

Table A3 – I: Comparison of eq. A3.12 from this work with Eq. 2 of Piet et. al.[9]

m, τ_R, τ_F	$CR_{m>0}$ Eq. 2 of Piet et al. (2009)	B_{FR}/L_{FR}	$CR_{m>0}$ eq. A3.12, $F_W = 0$
1.75%, 4, 2	1.17	0.56	1.19
1.75%, 4, 11	1.37	0.63	1.47

Sometimes it is convenient to solve for *m* in terms of *CR*. An iterative solution for $m > 0$, equivalent to $CR > 1 + F_w L_{FR}/B_{FR}$, can be found by gathering together higher order terms in *m*.

$$m_{>0} = \frac{\left(CR_{m>0} - 1\right)\left(B_{FR} / L_{FR}\right) - F_W - \left(1 + F_W\right)\left\{(1 + m\tau_{R,FR})(1+m)^{\tau_F} - \left[1 + m\left(\tau_{R,FR} + \tau_F\right)\right]\right\}}{\left(1 + F_W\right)\left(\tau_{R,FR} + \tau_F\right)} \qquad \text{Eq. A3.14}$$

Only a few iterations on *m* (starting with $m = 0$) are required for accurate convergence.

Equation A3.12 is somewhat different for the decaying case, $m < 0$. Solving for the situation where no extra TRU accumulates from decommissioning FRs, but rather the TRU unloaded from operation in year

n - τ_F - 1 is just what is needed to fuel the FRs in year n, and allowing for FR decommissioning to return fuel to the stock of FR-derived TRU (consistent with the derivation of Eq. A3.6), one arrives at a slightly different formula for CR:

$$CR_{m<0} = 1 + \frac{\left(1+F_w\right)(1+m)^{\tau_F+1} + \left(\tau_{R,FR} - 1\right)m - 1}{B_{FR} / L_{FR}} . \qquad \text{Eq. A3.15}$$

Of course in the limit $m \rightarrow 0$, $CR_{m<0} = CR_{m>0} = 1 + F_w L_{FR}/B_{FR}$.

The associated iterative solution for $m_{<0}$ is,

$$m_{<0} = \frac{\left(CR_{m<0} - 1\right)\left(B_{FR} / L_{FR}\right) - F_w - \left(1+F_w\right)\left[(1+m)^{\tau_F+1} - 1 - \left(\tau_F + 1\right)m\right]}{\tau_F + \tau_{R,FR} + F_w\left(\tau_F + 1\right)} . \qquad \text{Eq. A3.16}$$

Appendix 4: Equations for Stocks and Flows of Plutonium and Minor Actinides Associated with Fusion-Fission Hybrid Systems

A4.1 Transuranic Fueling Flow to Fusion-Fission Hybrids

The fusion-fission hybrid (FFH) system described by Stacey[10] uses a modest fusion system, producing 180–240 MW of fusion power, to drive a sub-critical fast reactor producing 3000 MWth output power by burning TRU. Since this system should be capable of producing ~1 GWe, the consumption of TRU is B_{FFH} = 1.096t/GWe-yr. There is no concomitant production of TRU, since no fertile material is included in the fuel loading. The calculated burn-up fraction[11] is $BF_{FFH} \sim 23.8$ percent, from which the total input load of TRU per GWe-yr can be calculated at L_{FFH} = 4.605t. The residence time of fuel in the system for this burnup is 2800 full-power days. Taking into account a reasonable duty factor this corresponds to $\tau_{R,FFH} \sim 9$ years.

A4.2 Transuranic Stock

Using the same simplified model for the TRU stock in FFH systems as in FRs, we have stock at mid-year in each FFH of $\tau_{R,FFH} L_{FFH} - BF_{FFH} L_{FFH} / 2$.

A4.3 Transuranic Unload Flow from Fusion-Fission Hybrids

The amount of TRU unloaded after 1 GWe-yr of production is just $L_{FFH}\left(1-BF_{FFH}\right)$. It is assumed that when an FFH is decommissioned, and another is commissioned, the net unload flow is not affected. When a net FFH system is decommissioned, its stock of TRU is returned to the pool of TRU.

A4.4 Processing Losses

As with FRs, processing losses are assumed to be one percent.

A4.5 Evolution Equations for Stocks and Flows

These equations are analogous with the FR equations of Appendix 3.

The total fueling flow needed for the FFHs on Jan 1 of year n is given by

$$F_{FFH}^{tot}(n)=L_{FFH}P_{FFH}(n)+\left(\tau_{R,FFH}-1\right)L_{FFH}\left[P_{FFH}(n)-P_{FFH}(n-1)\right] \qquad \text{Eq. A4.1}$$

whereas the total fueling flow available from prior operation of FFHs is given by

$$F_{FFH}^{FFH}(n)=\frac{L_{FFH}}{1+F_w}\left\{\begin{array}{l}\left(1-BF_{FFH}\right)P_{FFH}(n-\tau_F-1)\\+\left(\tau_{R,FFH}-1\right)Max\left[P_{FFH}(n-\tau_F-1)-P_{FFH}(n-\tau_F),0\right]\end{array}\right\} \qquad \text{Eq. A4.2}$$

including the source from net decommissioning of fusion-fission hybrids. (For simplicity we use the same symbol, τ_F, for the residence-time of the fuel in the reprocessing system as for FRs.) As with FRs, the fueling flow from the stock of LWR used nuclear fuel is just the difference between these.

To get to the full evolution equation, the additional contribution to TRU_{UNF} from LWRs must be included, as must loss to waste. Furthermore, the TRU_{UNF} must be reprocessed and fabricated into fuel, requiring an assumed period of one year. Taking these in account, we have, for the case of FFH systems, with no FR systems (we do not consider mixing the two):

$$\begin{aligned}&TRU_{UNF}(n)=TRU_{UNF}(n-1)\\&-\left(1+F_w\right)L_{FFH}\left\{P_{FFH}(n+1)+\left(\tau_{R,FFH}-1\right)Max\left[P_{FFH}(n+1)-P_{FFH}(n),0\right]\right\}\\&+L_{FFH}\left(1-BF_{FFH}\right)P_{FFH}(n-\tau_F)\\&+\left(\tau_{R,FFH}-1\right)L_{FFH}Max\left[P_{FFH}(n-\tau_F)-P_{FFH}(n-\tau_F+1),0\right]\\&+TRU_{p,LWR}P_{LWR}(n-1)+TRU_{LWRc}Max\left[P_{LWR}(n-1)-P_{LWR}(n),0\right]\end{aligned} \qquad \text{Eq. A4.3}$$

The total stock of TRU in the FFH system at any time, *n*, is given by the sum of the TRU in the FFHs, plus the total fueling for year *n +1,* multiplied by (1+F_w), plus all *FFH→FFH* fueling in process for other years.

$$\begin{aligned}&TRU_{FFHS}(n) = L_{FFH}\left[\tau_{R,FFH} - BF_{FFH}/2\right]P_{FR}(n)\\&+(1+F_w)L_{FFH}\left\{P_{FFH}(n+1)+(\tau_{R,FFH}-1)\left\{Max\left[P_{FFH}(n+1)-P_{FFH}(n),0\right]\right\}\right\}\\&+L_{FFH}(1-BF_{FFH})\sum_{m=n+1-\tau_F}^{n-1}P_{FFH}(m)\end{aligned} \quad \text{Eq. A4.4}$$

where we are not including the possibility of net reduction of FFHs over time, since that does not occur in the calculations shown here, nor are we allowing for the case of overproduction from FFHs, where the TRU unload from year n - τ_F - 1 is greater than (1+F_W) times the loading requirement for year *n,* also not a case considered here.

The flow of TRU to the waste stream is easily evaluated as F_w times the total flow to fueling FFHs:

$$TRU_W(n) = TRU_W(n-1) + F_w L_{FFH}\left\{P_{FFH}(n) + (\tau_{R,FFH}-1)\left[P_{FFH}(n) - P_{FFH}(n-1)\right]\right\} \quad \text{Eq. A4.5}$$

again assuming no net decommissioning of FFH systems during the time of calculation.

Conservation equations can be derived to provide numerical checks, analogous to those for FRs:

$$\begin{aligned}&TRU_{UNF}(N) - TRU_{UNF}(0) = TRU_{p,LWR}\sum_{n=0}^{N-1}P_{LWR}(n) + TRU_{LWRc}[P_{LWR}^{\max}(1,N) - P_{LWR}(N)]\\&-(1+F_w)L_{FFH}\left\{\sum_{n=1}^{N}P_{FFH}(n+1)+(\tau_{R,FFH}-1)P_{FFH}(N+1)\right\}+L_{FFH}(1-BF_{FFH})\sum_{1}^{N-\tau_F}P_{FR}(n)\end{aligned} \quad \text{Eq. 4.6}$$

and

$$\begin{aligned}&TRU_{FFHS}(N) = (1+F_w)L_{FFH}\left\{\sum_{n=1}^{N}P_{FFH}(n+1)+(\tau_{R,FFH}-1)P_{FFH}(N+1)\right\}\\&-L_{FFH}\left\{(1-BF_{FFH})\sum_{n=1}^{N-\tau_F}P_{FFH}(n)+\sum_{n=1}^{N-1}BF_{FFH}P_{FFH}(n)+0.5BF_{FFH}P_{FFH}(N)+F_w\left[\sum_{n=1}^{N}P_{FFH}(n)+(\tau_{R,FFH}-1)P_{FFH}(N)\right]\right\}\end{aligned}$$

Eq. 4.7

These and additional numerical checks confirm the self-consistency of the given solutions for FFH systems.

A4.6 Decay Rate of Fusion-Fission Hybrid Systems with Zero Transuranic Input

Since the FFH systems described here do not produce net positive amounts of TRU, there is no analogous case to the maximum growth rate without TRU input that was considered above for FRs. However there clearly is a decay rate of the FFH system in which individual FFH reactors are turned off as waste is burned, in just such a manner that the fuel emerging from the TRU stock at all times is just what is needed for each future year, allowing for FFH decommissioning to return fuel to the stock of FFH-derived TRU (consistent with the derivation of equation A4.3). Starting from equation A4.3, we can solve for BF_{FFH}:

$$BF_{FFH} = 1 - \left(1 + F_w\right)\left(1 + m\right)^{\tau_F + 1} - \left(\tau_{R,FFH} - 1\right)m \qquad \text{Eq. 4.8}$$

which has the physically intuitive limit as $m \to 0$ of $BF_{FFH} = -F_W$.

We can also form an iterative solution for m:

$$m = \frac{-BF_{FFH} - F_w - \left(1 + F_w\right)\left[\left(1 + m\right)^{\tau_F + 1} - 1 - m\left(\tau_F + 1\right)\right]}{\left(1 + F_w\right)\tau_F + \tau_{R,FFH} + F_w}. \qquad \text{Eq. 4.9}$$

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