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The Proliferation of Orbiting Fragments: a Simple Mathematical Model

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We present and discuss a simple mathematical model (two coupled, non-linear, first-order differential equations) for the future proliferation in low earth orbit of space debris that is created by high-velocity destructive collisions of small objects with artificial satellites or other "big" orbiting bodies. The model predicts that such collisional generation of fragments will be the dominant source of debris in a few decades. Subsequently, if the satellite launch rate remains comparable with the current one, a quasi-exponential growth of potential projectiles will cause the number of satellites to reach a peak (of the order of 10^4) in about 150 years and then to rapidly decline by about a factor of 10. Some alternative choices of model parameters (for example, to account for uncertainty in the projectile-to-target mass ratio required for breakup and the rate of future injection of material into orbit and intentional generation of debris) show that this evolution may be anticipated or delayed, but not qualitatively modified, unless ad hoc measures to avoid or limit collisions are adopted.

INTRODUCTION

At present the United States Space Command is tracking, using 29 radar and optical sensor facilities, about 7,000 orbiting objects, more that 95 percent of which exceed 10 centimeters in size, with a total mass of the order of $3 \cdot 10^6$ kilograms. Several investigations indicate the probable existence of 2,000 additional objects in the 10–20-centimeter range and some 50,000 objects in the 1–10-centimeter range.[1] These untrackable particles constitute a growing hazard for space operations.

About 23 percent of the catalogued space objects in earth orbit are pay-

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loads,* and among these only 5 percent are active satellites and probes; 10 percent are spent rocket bodies and 67 percent are debris, either fragments or objects intentionally discarded during satellite delivery or other operations.[1]

As of 1 January 1991 more than one half of the tracked objects, and almost all of the smaller debris, were the result of 100 discrete fragmentation events.[2] Twenty-four of these events had an unknown cause, 42 were deliberate, 32 were propulsion-related, and two were related to electrical or command failure.[2] Although it is difficult to demonstrate conclusively, it is believed that at least three of the events of unknown cause were unintentional collision-induced fragmentation events.[3]

During the last decade several design changes were introduced in order to avoid propulsion-related fragmentations and the operational release of debris, e.g., from pyrotechnic devices, yo-yo masses, sensor covers, and so on. On the other hand, the intentional destruction of satellites and rocket bodies has been justified over the years as needed to perform structural tests (as in the case of a Saturn-IVB upper stage ruptured by deliberate pressure build-up on 5 July 1966), to dispose of sensitive military satellites at the end of their operational life, and to conduct antisatellite and self-defense tests.[4] Three intentional breakups (including the above mentioned structural tests and two space weapon tests; see below) were performed by the United States, all 39 others being the result of Soviet space activity. Overall, at least 12 percent of these events were the result of Soviet and US antisatellite weapon (ASAT) tests; the remainder have been attributed mostly to the destruction of Soviet electronic ocean surveillance satellites or reconnaissance spacecraft unable to accomplish a controlled re-entry.[4]

ASAT tests have produced about 10 percent of all the catalogued breakup debris. The Soviet Union began a series of ASAT tests in 1968 and terminated them in 1982 (although a strange ASAT-like experiment was performed in 1985).[5] On the average, these events produced 50–100 catalogued space debris each.[4] The only US ASAT test resulting in target breakup was the successful fragmentation of the *Solwind* spacecraft by an air-launched miniature homing vehicle in September 1985. The impact produced more than 200 catalogued

* Ten of these carry on-board radioisotope thermoelectric generators and 33 deactivated nuclear reactors.

pieces of debris, most of which remained in orbit for several years. One year later, an SDI experiment involving the collision of a payload with its own upper stage resulted in the production of several hundred detectable pieces of debris; however, most re-entered within days, as planned, and only 18 were catalogued.[4]

The proliferation of small debris (detected or not) is having a growing impact on space operations, spacecraft design and maintenance, and overall project costs. For instance, when the space shuttle was designed, damage from artificial space debris was not considered a significant potential threat. However, during the first 30 shuttle missions, 27 windows in 18 flights showed some form of debris damage and 13 had to be replaced. Probably many of the windows were damaged during ascent or landing, but at least one event (the largest and only one examined in detail) has been associated with an orbital debris strike.[6]

For large space vehicles and structures planned to stay aloft years or decades (as the proposed *Freedom* space station), the probability of artificial debris impact is already troublesome and well in excess of that due to micrometeorites of equivalent mass. To make the matter worse, several hypervelocity impact experiments carried out in the laboratory have shown that collisions produce many more particles of size less than 10 centimeters, down to submillimeter flecks than low energy explosions.[7] The kinetic energy liberated by an impact of a particle of a few grams at the average relative velocity of 10 km s^{-1} (some 10^5 joules) corresponds to that of a hand-held grenade and can destroy an unshielded spacecraft[8] (or natural rocky body of similar mass).[9]

As a consequence, collisions between orbiting bodies could produce a cloud of many more objects in a sort of chain reaction, with a dramatic increase in the probability of new collisions. One estimate is that we are already in a situation where, even for zero launch rate of new spacecraft, the amount of space debris will continue to grow, eventually creating a debris belt around the earth similar to the asteroid belt.[10] Another estimate puts the population needed for triggering a chain reaction at about two or three times the current debris population, a situation that could be reached within 20–50 years at the present rate of space activity.[11] According to a recent report from the European Space Agency, "the self-sustained debris production by collisions is a long-term concern. It is however the most far-reaching threat which could terminate all space activities. This mechanism requires further careful study."[12]

In this paper we discuss a simple mathematical model of a debris chain reaction, which clearly shows its qualitative features and its sensitivity to various controllable parameters.

THE MATHEMATICAL MODEL

Our model is a close analogue of those used in ecology to describe the interaction between different living species or in the simulation of battles between opposing armies (the so-called Lanchester models of warfare). We assume that there are just two populations of orbiting bodies: N satellites, objects of cross section on the order of a few square meters and of mass of hundreds of kilograms; and n fragments, i.e., small bodies capable of causing catastrophic breakup when impacting a satellite. In our baseline case, the fragment population is characterized by typical sizes of order of 1 centimeter and masses of a few grams (as we mentioned above, at a collision velocity of 10 km s^{-1} a projectile-to-target mass ratio of 10^{-5} is enough to shatter most natural solid targets); however, we shall later analyze the case in which the satellites are more resistant to impacts. To study the interaction of these two populations, we adopt a collision rate proportional (through a constant coefficient x) to the product nN; when a collision occurs, one satellite is destroyed and new "collisional" fragments are created. Finally, we assume that every year A new satellites enter orbit (A should be interpreted as the difference between the number of satellites launched and the number re-entering the atmosphere); at the same time, "primary" orbiting fragments are created during the launch and satellite delivery operations.

These assumptions lead to the pair of first-order differential equations:

$$\frac{dN}{dt} = A - xnN \qquad (1)$$

$$\frac{dn}{dt} = \beta A + \alpha xnN \qquad (2)$$

A fairly similar model of the orbital-debris production process, based on a single differential equation, has been independently proposed in a recent paper by Talent;[13] in spite of somewhat different assumptions on the source and sink terms appearing in the equations, Talent's results are similar to those which can be inferred from equations 1 and 2, as discussed below. As for the values of the constants appearing in the equations and the initial conditions, we base our choices on some realistic (albeit very approximate) estimates. In our "standard" model we adopt:

(i) $A = 100$. This is the order of magnitude of the current rate of insertion of "big" objects (spacecraft, rockets) into orbit.

(ii) $x = 3 \cdot 10^{-10}$. This is consistent with the available estimates of collision rates in low earth orbit (about 10^{-5} y^{-1} m^{-2} for all objects larger than 1 centimeter; see reference 12, figure 4.2), taking into account that there are some 50,000 potential projectiles (see below). This value for x can be derived also by a simple particle-in-a-box computation for the collision rate: the number of collisions per unit cross section per year is of the order of the average collision velocity (≈ 10 km s^{-1}) divided by the volume of the circumterrestrial shell containing the population of orbiting bodies ($6 \cdot 10^8$ $km^2 \times 1{,}800$ kilometers $\approx 10^{12}$ km^3).

(iii) $\alpha = 10^4$. This follows from the typical mass distribution of fragments generated in hypervelocity impacts. With the cumulative number of fragments of mass greater than m proportional to about $1/m$, if the largest fragment is some tens of kilograms in mass, about 10^4 fragments should exceed a few grams. This is consistent with the available experimental evidence on these events (for example, see reference 9).

(iv) $\beta = 70$. Most of the "primary" fragments are generated in explosions during or after launches, involving rockets or second/third stages. The vast majority of the existing debris probably has this origin. If we assume an average of two unintentional explosions per year (from the 58 occurred since the beginning of the space age), with each of these events creating a few thousands of fragments of mass greater than 1 gram, the above estimate of β follows. We also recall that "primary" fragments include a number of mission-related objects delivered in orbit, like payload packing and separation devices, empty propellant tanks, shrouds, lens covers, etc.

(v) $N(0) = 2 \cdot 10^3$. This is consistent with the present abundance of catalogued payloads and spent rockets, taking into account that a few hundreds of them orbit above the densely populated, low-orbit shell between about 200 and 2,000 kilometer altitudes.

(vi) $n(0) = 5 \cdot 10^4$. As already mentioned, this is reasonable estimate of the present abundance of orbiting fragments larger than about 1 centimeter. Notice, however, that this estimate is uncertain by about a factor of 2.

Besides the "standard" model described above, we also consider four alternative models. In the first, the launch rate is assumed to grow linearly with time, in such a way to obtain twice the current launch rate in 50 years from now (i.e., $A' = A + 2t$, with t measured in years). In the second model, equation 2 will include an additional constant term B, meant to model the effects of intentional explosions or collisions, e.g., due to the continuation of ASAT weaponry tests. We assume $B = 10^4$, corresponding to about one explosion/collision per year. The third, "optimistic" model assumes that as a consequence of new provisions or agreements, no new "primary" fragments are inserted into orbit ($\beta = 0$). Finally, we test the hypothesis that the satellites can be collisionally disrupted only by projectiles somewhat larger and more massive than we have assumed so far (i.e., with typical sizes of a few centimeters and masses 10 grams): in this case we use the values $n(0) = 2 \cdot 10^4$, $\alpha = 10^3$, and $\beta = 20$.

RESULTS

We have numerically integrated equations 1 and 2, with the initial conditions described above, over a time span of 500 years or more in the future. Actually, the integration of one equation was sufficient, since it is easy to note that

$$\alpha N(t) + n(t) = (\alpha + \beta) A t + \alpha N(0) + n(0) \qquad (3)$$

Figures 1 and 2 show the results for the "standard" model. The following features appear to us as of particular relevance:

- At the beginning we have both $xnN \ll A$ and $xnN \ll \beta A$. Therefore collisions are not important, and the populations of both fragments and satellites increase almost linearly with time.

- When nN becomes larger than $\beta A/\alpha x \approx 2{\cdot}10^9$, which happens at about t = 40 years ($n \approx 4{\cdot}10^5$, $N \approx 6{\cdot}10^3$), the generation of collisional fragments exceeds that of "primary" ones, and the fragment growth becomes exponential. But as A is still less than nN, only a very small fraction of satellites are affected and the growth of satellites keeps almost linear (figure 1).

- At about t = 150 years, nN becomes comparable with $A/x \approx 3{\cdot}10^{11}$ ($N \approx 1.5{\cdot}10^4$, $n \approx 2{\cdot}10^7$). The abundance of satellites reaches its maximum, and afterwards it rapidly falls, implying that more satellites are shattered than launched. At the same time, the creation rate of new collisional fragments continues to grow until about t = 180 years (figure 2).

- Finally, at $t \approx$ 300 years, the abundance of satellites stabilizes around 10^3

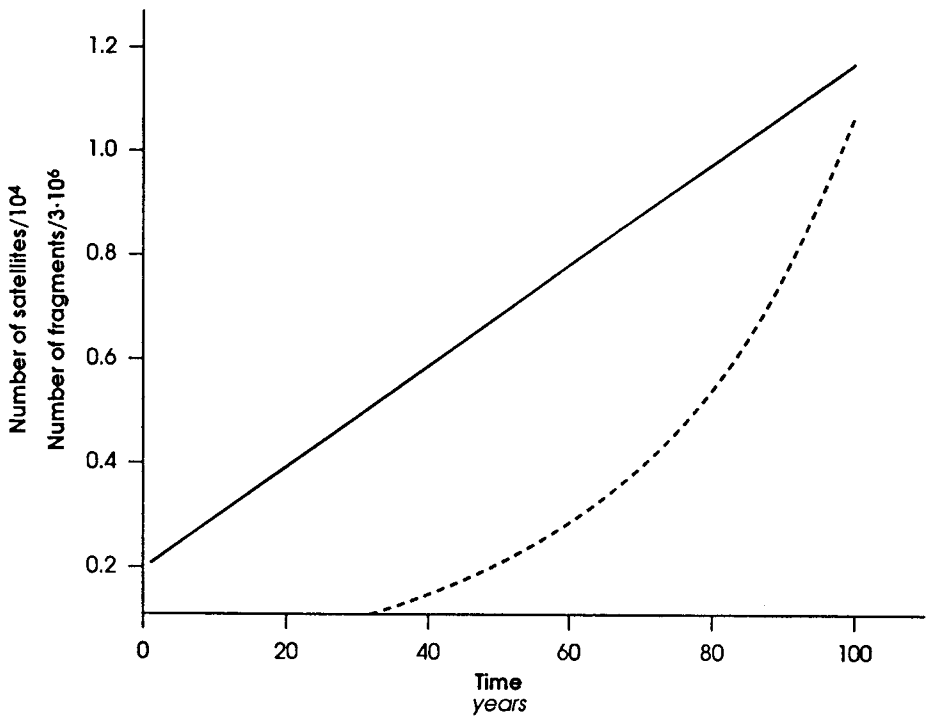

Figure 1: The number of "satellites" (full line) and "fragments" (dashed line) predicted by our mathematical model with the standard parameter choice over a time span of one century in the future. The vertical axis unit corresponds to 10^4 satellites and $3{\cdot}10^6$ fragments.

(but with a slow, continuing decline), and a quasi-steady state ensues in which the material launched into orbit is totally converted into fragments by collisions. At $t = 300$ years, the abundance of fragments reaches $3{\cdot}10^8$.

Figures 3 to 6 show the results of our four alternative models. Clearly, the main qualitative features remain unchanged. When the launch rate increases with time (figure 3), the initial population growth is quadratic instead of linear, and of course the peak value of N is higher: it exceeds $3{\cdot}10^4$ and is reached at $t = 145$ years; afterwards there is very rapid decline to a quasi-steady abundance of a few thousand satellites, while n exceeds 10^9. In the second model (figure 4), which includes the additional source of "primary" fragments (e.g., ASAT tests), the fragment proliferation grows exponentially from 10^5 to $2{\cdot}10^8$ (at which level the linear growth regime prevails again), while the number of satellites peaks at about $1.3{\cdot}10^4$ for $t = 130$ years; the final phase of slow

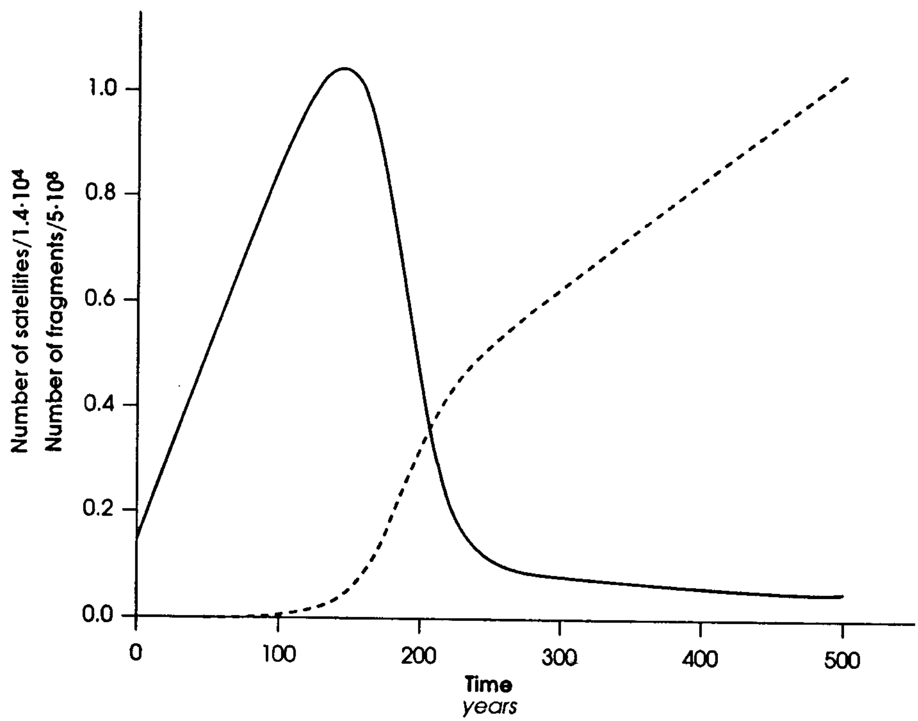

Figure 2: The same as in figure 2, but over a time span of 500 years in the future. Here the vertical axis unit corresponds to $1.4{\cdot}10^4$ satellites and $5{\cdot}10^8$ fragments.

decrease from about $N = 1.5 \cdot 10^3$ to $N = 10^3$ starts at about $t = 250$ years. The "optimistic" model (no new "primary" fragments, see figure 5) shows that the catastrophic decrease in the number of satellites is delayed until $t \approx 200$ years, but still is completed at $t \approx 300$ years (passing from $N = 1.8 \cdot 10^4$ to $N \approx 10^3$); the exponential proliferation of fragments is also delayed by a few decades. Finally, figure 6 shows the sensitivity of the results to the assumed size (or mass) of the fragments, which is equivalent to an assumption on how resistant a typical satellite is to collisional breakup; we recall that our parameter choice in this case corresponds roughly to a tenfold increase of the fragment mass. The quasi-linear increase of the fragments has in this case a much longer duration (≈ 200 years), the satellite number peak occurs for $t = 240$ years, $N = 3.8 \cdot 10^4$, $n = 8 \cdot 10^6$ (so that again $nN \approx A/x$), and then the decline lasts for several centuries. These results show that for a better quantitative modelling of

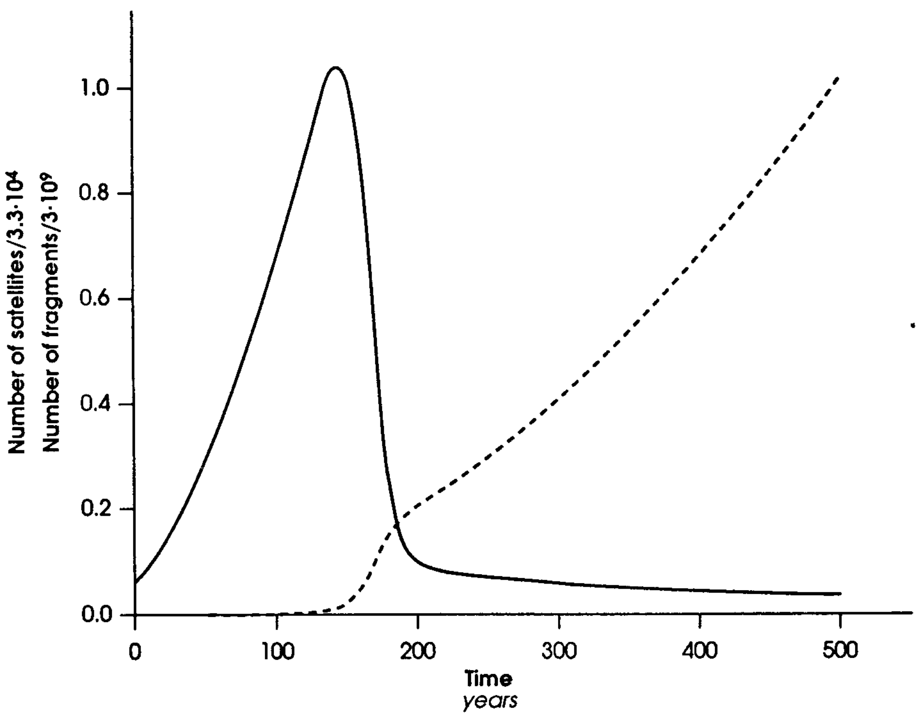

Figure 3: The same as in figure 2, but for the parameter choice corresponding to a linearly increasing number of launches. Here the vertical axis unit corresponds to $3.3 \cdot 10^4$ satellites and $3 \cdot 10^9$ fragments.

the whole process we need to know in a reliable way (possibly, from suitable hypervelocity impact experiments) the critical projectile-to-target mass ratio for catastrophic disruption of manmade space objects.

CONCLUSION

Of course the results described above are model dependent: for instance, the real mass distribution of earth-orbiting bodies (in particular, of the debris) is continuous and our assumption of just two discrete and interacting populations is artificial. From this point of view, more realistic models could be studied, in analogy with the work done on the collisional evolution of asteroids.[14] Moreover, one should take into account that the density and speed (hence the collision rate) of the earth-orbiting bodies is a function of height; the same

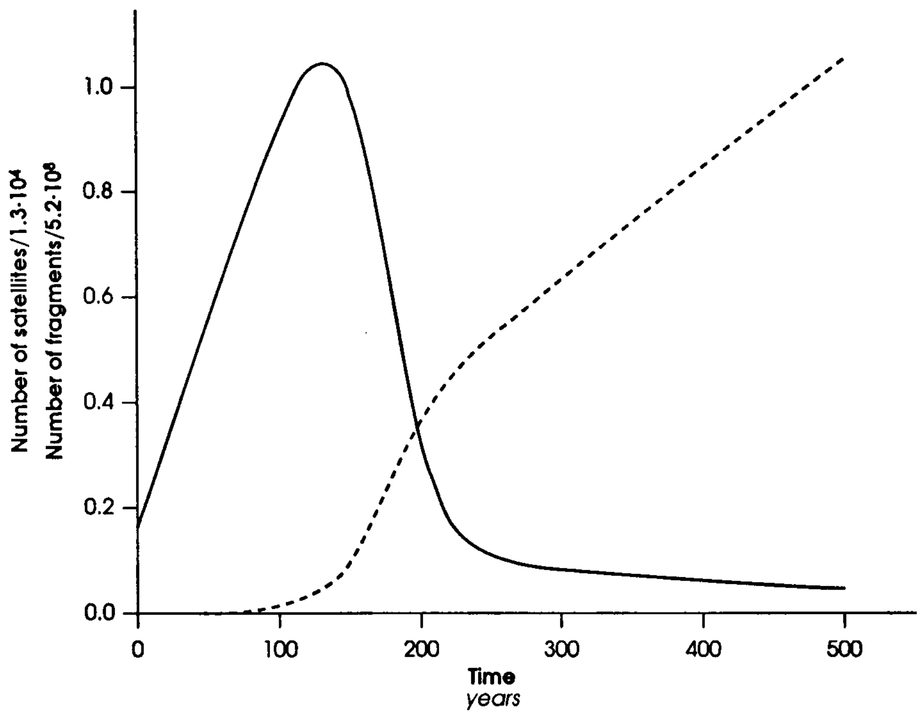

Figure 4: The same as in figure 2, but for the parameter choice corresponding to an increased injection rate of "primary" fragments (e.g., due to explosions or ASAT tests). Here the vertical axis unit corresponds to $1.3 \cdot 10^4$ satellites and $5.2 \cdot 10^8$ fragments.

holds for the removal rate due to atmospheric drag, which in addition depends on the area-to-mass ratio (i.e., size, shape and density) of the objects and is greatly affected by the 11-year solar activity cycle. However, we are confident that main features of our simple model correspond to the basic dynamics of the real process, and therefore provide a good semi-quantitative description of its evolution. This can be confirmed by comparing the results with those of other studies, based on different approaches to the problem.[10,13]

The main conclusion of this study is that there is an intrinsic limit in the abundance of satellites (and other "big" orbiting bodies) in low earth orbit, which is probably only one order of magnitude larger than the present population. The exponential growth of fragments, due to collisions, is likely to start a few decades in the future; later on, this will force the space agencies to decrease the overall rate of insertion of objects into orbit, in order to keep the

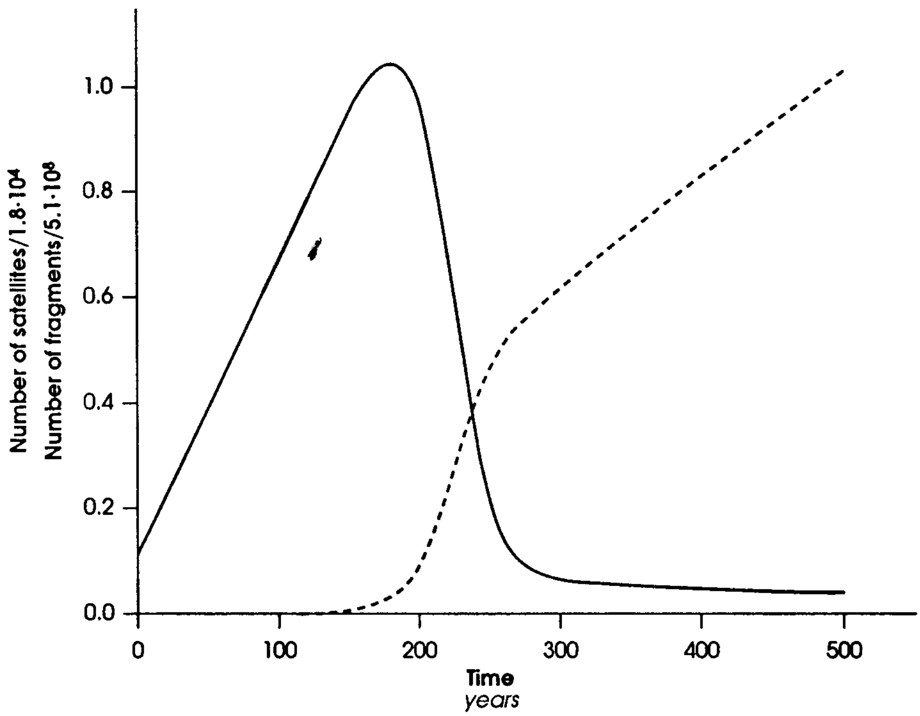

Figure 5: The same as in figure 2, but for the parameter choice corresponding to no future injection of "primary" fragments. Here the vertical axis unit corresponds to $1.8 \cdot 10^4$ satellites and $5.1 \cdot 10^8$ fragments.

total population of satellites within levels not much higher than the present one and to avoid a catastrophic decline of all space activities a few centuries in the future.

Avoiding in-orbit explosions, ASAT tests and, in general, the insertion into orbit (above a few hundred kilometers of height) of new "primary" fragments could delay the fragment proliferation problem by some decades. An important step, consistent with this policy and taken by NASA in 1982, has been the requirement for the venting of the unspent propellants and gases from the *Delta* upper stages, to prevent explosions due to the mixing of fuel residues. But possibly other, more radical measures will be required in the future to avert the final catastrophe discussed above. A schematic list of such measures includes: the disposal (through deorbiting) on non-operational potential targets and/or projectiles; their retrieval or removal by other techniques; the

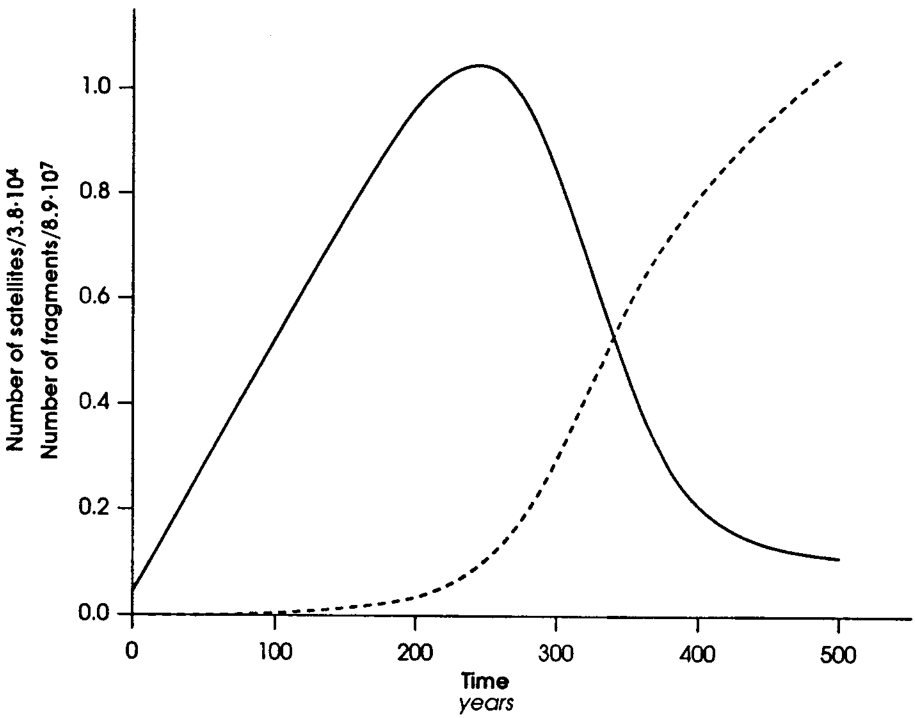

Figure 6: The same as in figure 2, but over a time span of 800 years in the future and assuming larger fragment sizes (i.e., more resistant satellites—see text). Here the vertical axis unit corresponds to $3.8 \cdot 10^4$ satellites and $8.9 \cdot 10^7$ fragments.

adoption of procedures for active collision avoidance; and the introduction of satellites more resistant to impacts (for example, by shielding or ad hoc design), and of smaller mass and cross-section. Some of these steps are currently under consideration or development, others are just research subjects.[15]

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