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An Atmospheric Limit on Nuclear-powered Microwave Weapons

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We present a simple model that predicts a limit to the microwave energy fluence (energy per unit area) that can propagate through the atmosphere and show that this limit falls within levels that can be shielded against. Within the relevant range of parameters, this model simplifies and extends results reported by Yee et al.[1] to show explicitly the dependence of the breakdown time on microwave field strength and altitude above sea level.

With continued nuclear-weapon testing, the possibility that the US and USSR could develop nuclear-explosion-powered microwave weapons capable of paralyzing mobile missiles or command systems has complicated arguments about comprehensive or low-yield-threshold test ban treaties.[2] In this note, we explore one physical effect that would limit the capabilities of such weapons—the rapid (subnanosecond) dielectric breakdown of air under the high electric fields present in intense microwave pulses.

This limit on the energy fluence deliverable in a microwave pulse, along with the difficulties of generating and focusing such a beam, leads us to conclude that nuclear-powered microwave weapons, like existing counterforce weapons, would not fundamentally alter the US–Soviet mutual nuclear-hostage relationship. Neither desire for a bloodlessly paralyzing "useable" weapon nor fear of a possible "gap" in progress toward the development of

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such a weapon should therefore be used to slow progress toward a nuclear test-ban treaty.[3]

ATMOSPHERIC SHIELDING OF INTENSE MICROWAVE PULSES

The physical effect to be described has the result that very powerful electromagnetic pulses in the atmosphere are "cut off" in the time required for the electric field in the leading edge of the pulse to break down the air into a dense plasma. In effect, the plasma created by the leading edge of the pulse builds up into a conductor that can block the passage of the remainder of the pulse if the plasma is thick enough. The required thickness is characterized relative to a local "skin depth" δ, the exponential attenuation length of an electromagnetic wave in such a plasma. Since δ varies inversely with the plasma electron density, it rapidly decreases as the electron plasma builds up. When the plasma becomes dense enough so that the local skin depth is short compared to the thickness of the ionized plasma, the transmission of the remaining portion of the pulse is blocked (see figure 1). Because the build-up of the ionization density is exponential with time, the cut-off time t_{co} occurs effectively as soon as the local plasma density at a distance ct_{co} behind the leading edge of a microwave pulse has built up to the point where the skin depth there equals the length of pulse that has already passed:*

$$\delta\,(z = -ct_{co}) \approx ct_{co} \tag{1}$$

Here $c \cong 3 \cdot 10^8$ m s^{-1} is the speed of light in vacuum, and the distance z is mea-

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* *Complete* cut-off would occur after just a few more doublings in electron density in addition to the 40–50 that are typically required to satisfy equation 1. One could also consider "erosion" of the pulse closer to its leading edge, where the plasma would not be dense enough to cut off the pulse but where microwave energy would still be transferred to the electrons and dissipated by collisions. For instance, if 60 ionization doublings occurred prior to t_{co}, then the local skin depth in the middle of the pulse would be about 2^{15} times longer than that at the rear, i.e., $2^{15} \cdot ct_{co}$ (the skin depth varies as the inverse *square root* of the electron density), or about 10 kilometers for $t_{co} = 1$ nanosecond. Since we have neglected this erosion effect, pulses propagating horizontally over distances of tens of kilometers would be somewhat shorter than our model predicts. For vertical propagation, however, the number of generations in the electron avalanche required to achieve the cut-off condition is only slightly more than that needed to produce a significant erosion effect over the narrow range of altitudes in which cut-off occurs, and the erosion effect therefore does not significantly shorten vertically propagating pulses.

sured back from the leading edge of the pulse.

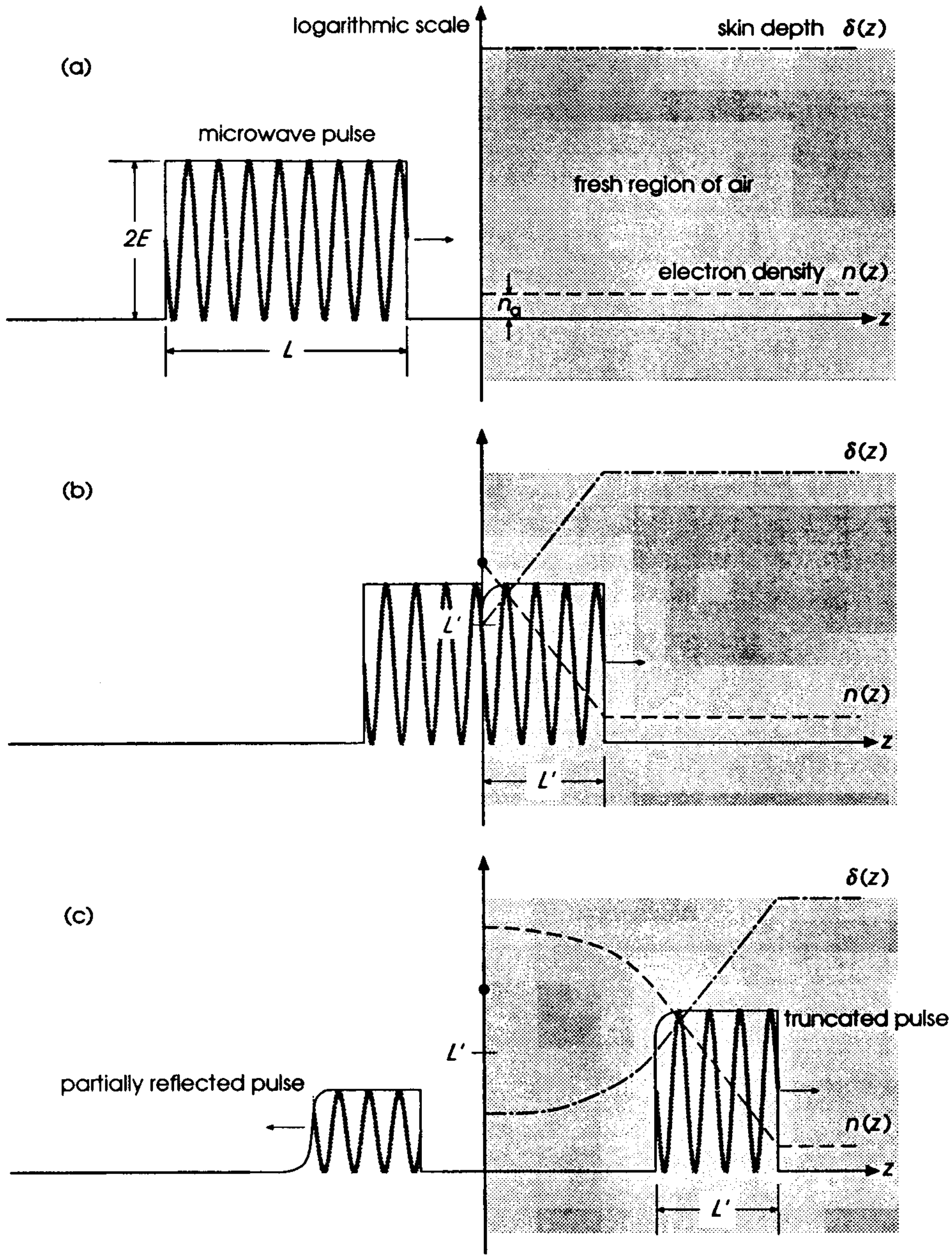
In appendix A, we use the simple model expressed by equation 1 to calculate the cut-off time as a function of both pulse electric-field strength and altitude, for cases in which the microwave frequency of the pulse is low compared to the collision frequencies causing the ionization. In the regimes where the model is well-justified, the results are found to be in remarkably good agreement with numerous experimental results as well as the more detailed numerical model of Yee et al., which does not make this low-frequency approximation.

We find that, for a field strength E of 1 MV m^{-1}, the onset of breakdown occurs just below 50 kilometers in altitude and ultimately results in a pulse no longer than 1–2 nanoseconds (see figure 2). Using the relationship for the electromagnetic energy flux or intensity I

$$I = \frac{E^2}{2\eta} \tag{2}$$

(where η = 377 for E in V m^{-1} and I in W m^{-2}) this corresponds to a microwave power flux of about 10^9 W m^{-2} and a nanosecond fluence of 1 to 2 J m^{-2}.* In theory, stronger pulses can deliver somewhat more energy, but this can only occur if the *rise-time* of the electric field at the leading edge of the pulse is considerably shorter than 1 nanosecond. Such fast time-scales would be exceedingly difficult to achieve with a nuclear explosive-powered device, because its radiation release and hydrodynamic time-scales are on the order of 10 nanoseconds or more.Furthermore, since stronger pulses are cut off more rapidly (t_{co} at the altitude of maximum cut-off scales roughly as the inverse of the electric field), maximum energy *fluences* through the atmosphere would scale approximately only as the first power of the electric field rather than as the square. Thus *no matter how powerful or directional the microwave generator*, the energy in the pulse that can be propagated to the lower atmosphere is quite limited.

* The fluence is the energy flux integrated over its duration. In our low-frequency model, E is taken to be the maximum electric field of a sinusoidal wave. $\eta \equiv (c\varepsilon_0)^{-1}$, where $\varepsilon_0 \cong 8.85 \cdot 10^{-12}$ F m^{-1} is the permittivity of free space.


(a)
logarithmic scale
skin depth δ(z)
microwave pulse
fresh region of air
2E
electron density n(z)
n_a
z
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(b)
δ(z)
L'
n(z)
z
L'
(c)
δ(z)
truncated pulse
L'
partially reflected pulse
n(z)
z
L'

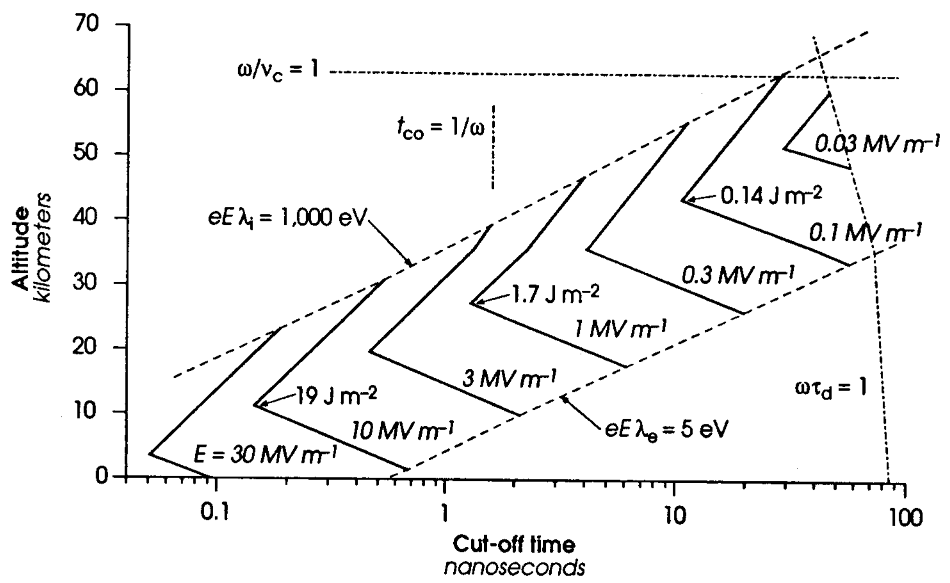

Figure 2: Cut-off time t_{co} plotted against altitude for microwave pulses with various maximum electric-field strengths E. The amount of energy fluence remaining in the pulse is given at several of the "knees" of the curves. The boundaries of validity imposed by the model's assumptions are indicated for a microwave frequency $\omega/2\pi$ = 0.1 gigahertz. (The curves for 3 gigahertz would be only about a line's width to the right of those plotted, but the low-frequency approximation would then be justified only for cut-off times less than about 3 nanoseconds or, equivalently, for electric fields E greater than 1 MV m^{-1}.) The dashed boundaries of reliability are explained in the text.

Figure 1: Schematic diagram illustrating the mechanism of plasma build-up and cut-off as an idealized intense electromagnetic pulse moves into a fresh region of only very weakly ionized air. (The vertical scale for skin depth and electron-density is logarithmic.) (a) A pulse of length L and field strength E approaches the new region of air, whose ambient electron density is n_a. (b) As a result of dielectric breakdown by the high electric field, a plasma with electron density n builds up behind the leading edge of the pulse and the local skin depth δ decreases. (c) The front portion of the pulse continues to propagates while the rear portion is partially absorbed and partially reflected. The cut-off occurs almost immediately after the local skin depth (at a distance $L' = ct_{co}$ behind the leading edge of the pulse) becomes short enough to equal the length of pulse L' that got through before the ionization had a chance to build up.

DAMAGE THRESHOLDS FROM MICROWAVE PULSES

As is well known, electronic components are many orders of magnitude more susceptible to EMP damage than electrical machinery or ordinary materials. In general, electronics can be damaged by submicrosecond pulses of microwaves with component-absorbed energy in the range of only 10^{-6} joules (sensitive microwave diodes), to 10^{-3} joules (high-power transistors and rectifier diodes).[4] Computer chips fall somewhere in the middle of this range (see table 1)[5]

For a microwave pulse with a characteristic wavelength λ_p, components might be expected to absorb on the order of $\lambda_p^2/4\pi$ times the energy fluence.[6]

Table 1: Microwave damage thresholds for electronics[a]

	E.J. Lerner[b]	Ricketts et al.[c]	Antinone[d]
		kilojoules	
Motors and Transformers	10 - 4,000	-	-
		millijoules	
Vacuum tubes	1 -10,000	-	-
Relays	2 - 1,000	2 -100	-
Resistors	1 - 1,000	~10	-
Rectifier and Zener diodes	0.5 - 300	~0.5 - 1	0.3 - 100
Medium and high-power transistors	0.1 - 100	~1	0.1 - 10
Ge and low-power transistors	0.003- 10	0.02- 1	0.003- 1
		microjoules	
Switching diodes	-	70 -100	-
Integrated circuits	0.1 - 1,000	~10	3 -1,000[e]
Microwave (mixer) diodes	0.1 - 100	0.7 - 12	0.2- 20

a. Columns 2 and 3 are specific threshold energies for 1-microsecond pulses, which would be expected to be only slightly higher than thresholds for much shorter pulses.

b. Eric J. Lerner, "Electromagnetic Pulses: Potential Crippler," *IEEE Spectrum*, May 1981, p.43.

c. L.W. Ricketts, J.E. Bridges, and J. Miletta, *EMP Radiation and Protective Techniques*, (New York: Wiley, 1976), pp.28–29, 76.

d. Robert J. Antinone, "How to Prevent Circuit Zapping," *IEEE Spectrum*, April 1987, p.38.

e. TTL integrated circuits are believed to have a damage threshold from 3–100 microjoules, CMOS from 10–1,000 microjoules, and "linear" integrated circuits from 30–1,000 microjoules.

For subnanosecond pulses, $\lambda_p \le 0.3$ meters and $\lambda_p^2/4\pi \le 0.01$ m^2. In the absence of shielding at the component, atmospheric transmission of 1 J m^{-2} would therefore translate into a limit on component-absorbed energy of the order of 10^{-2} joules or less. A subnanosecond pulse with a fluence of 1 J m^{-2}—which is two to three orders of magnitude greater than the fluences generated by normal high-altitude EMPs from high-yield nuclear explosions[7]—might therefore be a considerable threat to front-end antenna circuitry and certain types of *un*shielded electronics.*

Depending upon details of geometry, component orientation, and wave polarization, however, circuitry enclosed by metal structures with only seams, apertures, or small conduits to the outside world could have a dramatically decreased susceptibility to such pulses (see figure 3). Shielding factors of 10^{-3} to 10^{-5} in power are not difficult to achieve in practice.[8] Shielding against EMP is a well-studied art, and although tighter boxes and faster acting circuitry may be necessary, many of the same principles used to protect circuits against nearby lightning bolts or high-altitude EMP would also be applicable to protection against pulses from a nuclear-explosion-powered microwave generator. Even front-end receiver circuitry can be shielded to a high degree using a combination of fast-acting solid-state diodes, which can have turn-on times of less than 1 nanosecond, and delay lines to switch out unwanted spikes.[9] Electronics inside mobile missiles, aircraft, or command posts could therefore probably be shielded against pulses from nuclear-explosion-powered microwave generators. Certainly, an attacker should not have a high level of confidence that such weapon systems could be incapacitated at the level of fluence that could be transmitted through the atmosphere.

MODEL RESULTS

The cut-off times, calculated in appendix A, are plotted in figure 2 as a function of altitude and electric field. In this figure, a microwave pulse entering

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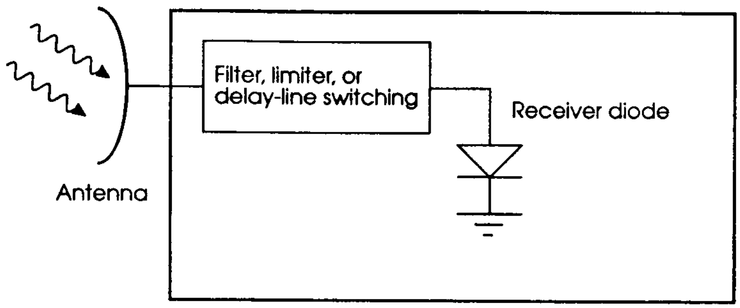
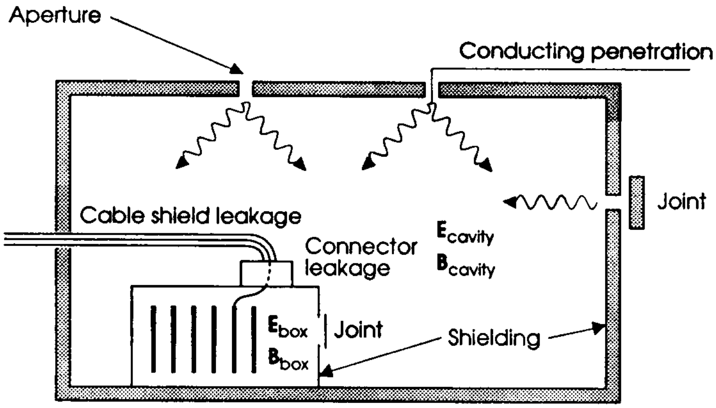
* For comparison, microwave heating of tissue and skin-burns occur only at very high fluences, in the range $(0.2–1)\cdot 10^6$ J m^{-2}, and melting or cracking of metals requires as much as $(10–100)\cdot 10^6$ J m^{-2}. See H. Keith Florig, "The Future Battlefield: A Blast of Gigawatts?," *IEEE Spectrum*, **25**, 3 (March 1988), p.53.



FRONT-DOOR COUPLING



BACK-DOOR COUPLING

Figure 3: Schematic diagram illustrating the mechanisms of "front- and back-door coupling" of electromagnetic pulses to electronic circuits. "Front-door" coupling involves the pulse being picked up by an exposed element of receiving circuitry (the "antenna" here). Circuit protection is provided by a filter or by switching out such pulses using a delay line. "Back-door" coupling involves the pulse leaking through protective shielding that often surrounds the circuitry. (Adapted from Florig, "High-Power Microwave Coupling," p.105.)

the atmosphere from space—if it were long enough—would first cause breakdown somewhat above and to the right of the top of the angular curve corresponding to its electric field.* It would then become progressively shorter, as described by the curve, until it encountered the knee of the curve.† Below that altitude, it would be shortened little if at all during the remainder of its descent to the earth's surface.

Above and below the indicated domain of validity, the curves for t_{co} would eventually bend to the right, and cut-off would take longer than what would be predicted by simply extending the angular curves in figure 2. Far below the bottom boundary, cut-off would eventually fail to take place at all, because the electrons would undergo so many elastic and ultimately inelastic collisions that they would not be accelerated to ionizing energies, and the true curves would become horizontal.‡

As figure 2 indicates, a pulse with a power of 10^{13} W m^{-2} ($\approx$ 100 MV m^{-1}) would cut itself off near the earth in about 0.02 nanoseconds, transmitting up to 200 J m^{-2} at centimeter wavelengths. However, this fluence would only get through if the *initial rise-time* of the pulse was of the order of 0.02 nanoseconds or less—an implausibly short time-constant for a nuclear-powered device. Therefore, it is more likely that the pulse would be cut off at lower values of its electric field and transmit much less overall energy.

In figure 4, we compare the predictions of our breakdown model with a

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* The angular curves resulting from the model in figure 2 are, of course, approximations for what in reality would look more like parabolas opening to the right. For a downward-propagating pulse, one would therefore expect the most rapid decrease in pulse-length (per kilometer of altitude) to occur over a very narrow range of altitudes near the top of each angular curve—where they meet the boundary $eE\lambda_i$ = 1,000 eV. This, in fact, is exactly what is seen in numerical simulations of such propagation. Cf. Yee et al., "Theory of Intense Electromagnetic Pulse Propagation," p.1242 (figure 5).

† This knee occurs at the altitude where the mean free path for electrons initially at rest in the atmosphere is such that they gain just the right amount of kinetic energy from a given electric field between each collision (about 20 electron volts) so as to cause ionizations most quickly. See appendix A, equations A-12a and A-12b.

‡ In the low-field low-frequency regime, experiments have shown that cut-off does not take place at all (the cut-off time goes to infinity) when the electric field is so weak that electrons gain only about 1 eV of energy between elastic collisions, i.e., $eE\lambda_e \approx 1$ eV. (See Yee et al., figure 5.)

For very high field strengths of around 30,000 MV m^{-1} (corresponding to a pulse energy flux of about 10^{18} W m^{-2}), far to the *left* of the curves shown in figure 2, electrons would begin to tunnel out of air molecules and the ionization would occur much faster than even this model predicts, thus providing another constraint on the total electromagnetic energy flux that can be propagated through air.

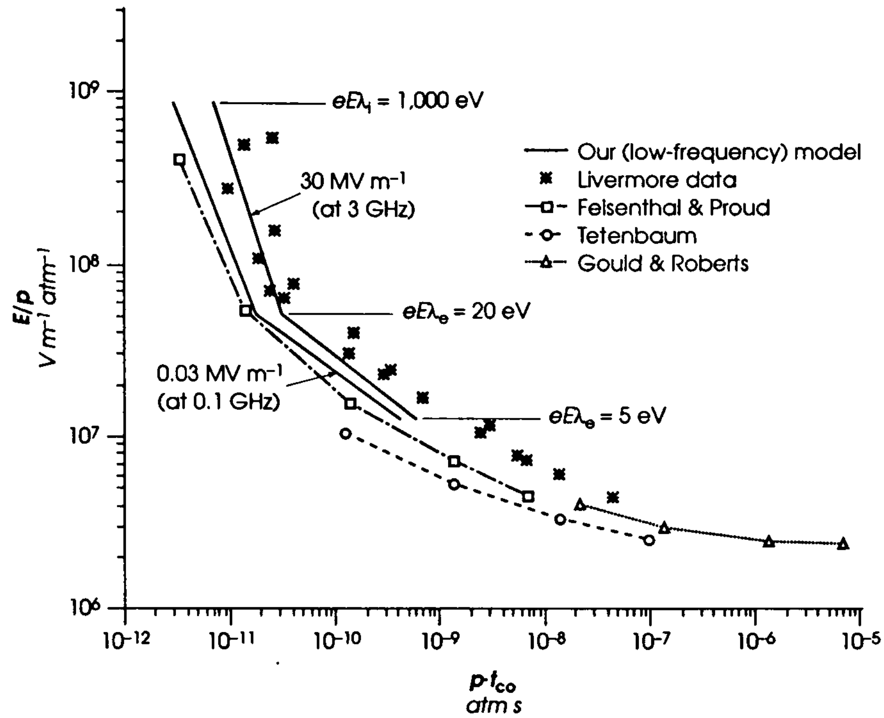

Figure 4: Universal plot of the electric field of a microwave pulse (normalized by the pressure) vs. the product of pressure and cut-off time—for our model and various experimental results. The indicated values of electron energy (5, 20, and 1,000 eV) at points along the curves correspond to three points along each constant-electric-field curve plotted in figure 2: 5 and 1,000 eV correspond to the electron energies achieved in one elastic or ionizing mean free path, respectively, at the lowest and highest altitudes justified by the model's assumptions; 20 eV is the energy gained in one elastic mean free path at the knee of the curves. For one-mean free path electron energies below 5 eV, the cut-off times are longer than would be predicted by simply extending the lower straight line predicted by our model. The difference between the lower (0.03 MV m^{-1}; 0.1 gigahertz) and upper (30 MV m^{-1}; 3 gigahertz) curves is primarily due to the large variation in the ambient "seed" electron density at the altitudes where these curves apply and, to a lesser extent, the difference in microwave frequency. (References for experimental data points: P. Felsenthal and J.M. Proud, *Phys. Rev.*, **139**, p.A1796 (1965); S.F. Tetenbaum, A.D. MacDonald, and H.W. Bandel, *J. Appl. Phys.*, **42**, p.5861 (1971); and L. Gould and L.W. Roberts, "Breakdown of Air at Microwave Frequencies," *J. Appl. Phys.*, **28**, p.1167 (1956). Livermore data is from Yee et al., p.1243 (figure 9). All experimental data were obtained at 2.8 gigahertz.)

number of experimental results, which fall near a "universal curve" when plotted using the composite variables E/p and pt_{co}, where p is the atmospheric pressure. The agreement is quite good over a wide range of electric fields and pressures. Indeed, we find that our model predicts the correct behavior of the cut-off time even somewhat into the regimes where the low-frequency approximation or the sufficient-energy assumption ($eE\lambda_e \geq 5$ eV) are not strictly satisfied.*

CONCLUSIONS

The predictions of our model can be used to arrive at bounds on the potential fluences of pulses produced by microwave generators exploding both within and above the atmosphere.

Explosion of a Microwave Generator in the Atmosphere

Even if a pulse with a strength of 200 J m^{-2} were short enough to be able to survive atmospheric breakdown at 10 meters from a detonation—and we have not been able to come up with any mechanism that would produce a microwave pulse short enough to have such a fluence—the r^{-2} fall-off of the energy flux would result in that pulse having a strength of only 0.02 J m^{-2} by the time it had propagated 1 kilometer, where other nuclear effects would still be extremely lethal, and 0.0002 J m^{-2} by 10 kilometers. Therefore, nuclear-powered microwave generators of potential military significance would require that their microwave flux be generated in space and directed downward, so that the energy could spread out over a large area before it entered the atmosphere.

Exo-atmospheric Directed-microwave Weapons

An exo-atmospheric nuclear-explosive–powered EMP weapon might be able to produce a directed microwave pulse by sending a beam of neutrons or gamma

* For instance, the low-frequency approximation ($\omega\tau_d < 1$) would not apply to the curve corresponding to an electric field $E = 0.03$ MV m^{-1} at a frequency of 3 gigahertz. But the predictions of our model—if plotted—would fall between the two (angular) curves from the model that are plotted in figure 4, and would thus still fit the experimental results shown there. One can also see from the figure that extending the curves below the point where $eE\lambda_e = 5$ eV would also fit the data fairly well down to a value of about 2–3 eV.

rays into the atmosphere using a nuclear shaped-charge, generating an EMP at altitudes between 20 and 40 kilometers that would then propagate down to the ground,[10] or it might generate and focus a microwave beam using the device itself.[11]

If 0.1-radian directivity and 10^{-5} to 10^{-3} efficiency are assumed, then a 1-kiloton device exploded at an altitude of 200 kilometers could illuminate a 20-kilometer diameter circle on the ground with an average fluence of 0.1–10 J m^{-2} in the absence of atmospheric breakdown or attenuation.[12] One might guess that, with a more efficient design, a higher-yield device, or by exploding the device at lower altitude, higher fluences might be delivered to the earth's surface. However, according to our model, unless the energy could be compressed into a pulse much shorter than 1 nanosecond—which is implausible since the radiation release and hydrodynamic expansion from fission and fusion explosive processes occur over time-scales of about 10 nanoseconds—air breakdown occurring within 1 nanosecond at electric fields around 1–3 MV m^{-1} would limit the fluence of any such electromagnetic pulse propagating down through the atmosphere to considerably *less than* 10 J m^{-2}. If the rise-time were several nanoseconds, the fluence that passed through the atmosphere would be less than 1 J m^{-2} (see figure 2). Thus, 1–10 J m^{-2} is a rough upper limit on the microwave fluence that can propagate through the atmosphere from any plausible nuclear device, independent of the altitude of burst, efficiency, yield, frequency, or directivity.*

Since such a fluence could require precautionary measures or additional expense for shielding beyond that already incorporated for protection against ordinary EMP effects and conventionally-produced microwaves,[13] nuclear microwave devices aimed at achieving these levels might remain of at least marginal interest for some strategic planners and weapon designers and could contribute to the continuation of the nuclear arms race. However, their use would bring the same risk of escalation to all-out nuclear war as any other

* To produce higher fluences one would have to create very long pulses with much lower (non-ionizing) electric fields, whose accumulated fluence over microseconds or milliseconds could then exceed these limits. However, such long pulses would be much easier to shield against—especially if they were low-frequency (long-wavelength) as well—and would be less lethal to electronics even if they penetrated the shielding because of heat dissipation over these time-scales.

nuclear weapon, without having a decisive effect. For all practical purposes, they should therefore be considered no more useable nor any less dangerous than other nuclear weapons.

ACKNOWLEDGEMENTS

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APPENDIX A: THE ATMOSPHERIC BREAKDOWN MECHANISM AND t_{co}

We describe here a model of atmospheric dielectric breakdown by electromagnetic pulses in which the microwave frequencies are assumed to be small in comparison to electron collision frequencies. This low-frequency model sufficiently covers the regime of interest for our purposes.

Because of the effects of cosmic rays, natural radioactivity, and (at higher altitudes) ultraviolet rays, the atmosphere contains an ambient density of free electrons n_a that increases very rapidly in the lower region of the ionosphere:[14]

$$n_a \approx 100 \text{ m}^{-3} \text{ below 36 kilometers}$$
$$n_a \approx 10^6 (h/60 \text{ km})^{18} \text{ m}^{-3}, \text{ for } h = 36 \text{ to } 100 \text{ kilometers.} \quad \text{(A-1)}$$

If the electric field of a microwave pulse accelerates one of these electrons to an energy U_i sufficient to ionize an air molecule ($U_i \approx 10$ eV* on average for air), then it can collisionally eject another electron. If the colliding electron's energy is high enough (above 40 eV), then such an ionizing collision is more probable than an elastic collision. In this case, both the initial and ejected electrons can quickly cause additional ionizations, leading to an exponential avalanche that creates a plasma of conducting (free) electrons.

Such an exponential electron avalanche can be characterized by a doubling time τ_d:

$$n = n_a 2^{t/\tau_d} \quad \text{(A-2)}$$

The doubling time depends upon the strength of the accelerating electric field and the "mean free path" of the electrons between collisions with air molecules.

* The electron volt (eV) is a unit of energy equal to $1.6 \cdot 10^{-19}$ joules. An electron would gain 1 eV of kinetic energy if accelerated in a vacuum over a distance of 1 meter by a constant electric field of 1 V m^{-1}.

Electron Acceleration

The mean free path λ is given by

$$\lambda = \frac{1}{N\sigma} \tag{A-3}$$

where σ is the (energy-dependent) total equivalent "cross section" or target area represented by an average air molecule, and N is the number density of air molecules. N decreases exponentially with height h and can be approximated up to about 100 kilometers above sea level by:[15]

$$N \approx 2.7 \cdot 10^{25} \exp\left(\frac{-h}{7\ \text{km}}\right) \text{m}^{-3} \tag{A-4}$$

Non-ionizing or *elastic* scattering dominates in air at electron energies U below about 10 eV and has an associated cross section[16]

$$\sigma_e \approx 10^{-19}\ \text{m}^2 \quad (U \le 10\ \text{eV}) \tag{A-5}$$

Ionizing collisions dominate above an electron energy of about 40 eV* with a roughly constant cross section thereafter up to an energy of about 1,000 eV having a value[17]

$$\sigma_i \approx 0.3 \cdot 10^{-19}\ \text{m}^2 (40 \le U \le 1,000\ \text{eV}) \tag{A-6}$$

The elastic and ionization mean free paths in these regimes are therefore

$$\lambda_e \approx 0.4 \cdot 10^{-6} \exp\left(\frac{h}{7\ \text{km}}\right) \text{meters}, \quad (U \le 10\ \text{eV}) \tag{A-7a}$$

$$\lambda_i \approx 1.2 \cdot 10^{-6} \exp\left(\frac{h}{7\ \text{km}}\right) \text{meters}, \quad (40 \le U \le 1,000\ \text{eV}) \tag{A-7b}$$

If the electric field strength E in the microwave pulse is high enough so that an electron will be accelerated to at least 20 eV in an elastic mean free path, i.e.,

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* $\sigma_i/\sigma_e \approx$ (1/30, 1/3, 1, 3) at U = (10, 20, 40, 80) eV, respectively. σ_i decreases by a factor of about 3 between 80 eV and 1,000 eV. The value we use for the wider range (40 eV to 1,000 eV) is an intermediate one.

$$E\lambda_e \geq 20 \text{ V or } E \geq 50 \exp\left(\frac{-h}{7 \text{ km}}\right) \text{MV m}^{-1} \tag{A-8}$$

then collisions in an electron avalanche are likely to be ionizing rather than elastic.* If the time between collisions is short compared to microwave frequency—i.e., $\omega\tau_d < 1$, the low-frequency approximation—then the time between electron doublings for this high-field case $\tau_d^{(0)}$ (in which electrons undergo no elastic collisions) may be approximated by the time required for an accelerating electron, starting at rest, to travel an ionization mean free path λ_i:†

$$\tau_d^{(0)} = \sqrt{\frac{2m_e\lambda_i}{eE}} \cong \frac{3.7 \cdot 10^{-9}}{\sqrt{E}} \exp\left(\frac{h}{14 \text{ km}}\right) \text{seconds } (E\lambda_e \geq 20 \text{ V}) \tag{A-9}$$

where we have used equation A-7b and E is in V m^{-1}.‡ At *very* high electric fields ($E\lambda_i >$ 1,000 V), the ionization cross section decreases appreciably and doubling times would be longer than that given by the right-hand side of equation A-9. Thus equation A-9 applies only up to $E\lambda_i \approx 1{,}000$ V.

At lower field strengths ($E\lambda_e < 20$ V), the doubling time between ionizations will be lengthened not only because of slower electron acceleration but also because elastic collisions interrupt the acceleration process and randomize the electron velocity direction. In a constant electric field E, the average time needed for an electron initially at rest to gain an energy U by a collisional random walk in velocity-space is

$$\tau_U^{\text{rand}} \cong \sqrt{\frac{2m_e\lambda_e}{eE}} \cdot \left(\frac{4}{3}\left[\frac{U}{eE\lambda_e}\right]^{3/2} - \frac{1}{3}\right) (\text{for } U \geq eE\lambda_e) \tag{A-10}$$

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* Although an electron accelerating from rest is more likely to be scattered elastically than cause an ionization while its energy is less than 20 eV, most electrons in an ionizing cascade will *begin* with energies of 10 eV or more, since most ionizations will occur at energies above 20 eV and lose only about 10 eV of energy to the ionization. Therefore, the condition in equation A-8 is sufficient for the avalanche to be dominated by ionizing collisions.

† Since most electrons created by ionizing collisions do not start at rest, this estimate of ionization time is an upper bound.

‡ For electric fields just satisfying the condition $E\lambda_e = 20$ V in equation A-9, the doubling time is given by $\tau_d^{(0)} \cong 5 \cdot 10^{-4} \exp(h/[7 \text{ km}])$ nanoseconds. (This condition is satisfied at the knees of the curves in figure 2.) This means that the low-frequency approximation $\omega\tau_d < 1$ would hold at these points for microwave frequencies up to about 15 gigahertz at an altitude of 20 kilometers, and up to 50 megahertz at 60 kilometers. For higher electric fields at any given altitude, the low-frequency approximation would be satisfied for even higher frequencies.

where λ_e is again the mean free path between elastic collisions.* If we set $\tau_U^{rand} = \tau_d^{(0)}$ (from equation A-9) at the transitional field strength of $E = 20$ V/λ_e—to obtain a continuous transition of the doubling time between the low-field random collisional regime and the high-field direct route to an ionization—we find that $U = 26.8$ eV is the appropriate value to use in equation A-10.†

Now, if the electric field is so weak that three or more collisions occur as an electron passes through the energy range from 9–11 eV—where there is a strong probability that the electron will lose its energy to an internal resonant excitation or dissociation of diatomic oxygen or nitrogen[18]—then most electrons would not accelerate to an ionizing energy because the inelastic collisions would keep draining that energy away.‡ From equation A-10, we can derive an expression relating the number of collisions undergone by an electron before reaching its final energy#

$$\text{Number of elastic collisions} = 2\left(\frac{U}{eE\lambda_e}\right)^2 - 1 \qquad \text{(A-11)}$$

Using equation A-11, we find that an electron will undergo three collisions while in the energy range 9–11 eV when $eE\lambda_e \cong 5$ eV. We therefore take $eE\lambda_e = 5$ eV as a lower bound on the range of electric field strengths over which our model is valid.

The doubling times in our model are summarized from equations A-9 and A-10 as follows:

For 20 V/$\lambda_e \leq E \leq$ 1,000 V/λ_i, ionization occurs at the first collision (after an electron traverses λ_i), and

* After an elastic collision randomizes the velocity direction to an average of 90° to the electric field, the average energy gained before the *next* collision-time $\Delta t_{coll} \cong \lambda_e(2U/m_e)^{-1/2}$ is $\Delta U = m_e(\Delta v)^2/2 = (eE\Delta t_{coll})^2/2m_e$. Setting $\Delta U = dU/dt \cdot \Delta t_{coll}$, substituting for Δt_{coll}, and integrating over U from the first collision, when $U = eE\lambda_e$ and $t = (2m_e\lambda_e/eE)^{1/2}$, to the final energy gives the result in equation A-10. This can be compared with the time for an electron to accelerate *without collisions* from rest to an energy U: $\tau_U^{no\text{-}coll} = (2m_e\lambda_e/eE)^{1/2}(U/eE\lambda_e)^{1/2}$. ($e/m_e \cong 1.76 \cdot 10^{11}$ C kg^{-1}.)

† In reality, an accelerating electron would ionize an atom on average as soon as it has traveled the integrated λ_i appropriate to its energy U, where λ_i decreases with energy up to $U \cong 40$ eV. Our criterion of random-walking to $U = 26.8$ eV before ionization, rather than traveling an energy-dependent λ_i, is a conservative approximation, erring on the side of predicting cut-off times slightly too long. At the transitional field in our model ($E\lambda_e = 20$ V, or at the knees of the curves), electrons take the same time to cause an ionization either by reaching 26.8 eV after about 2.6 collisions of length λ_e, or by reaching 60 eV after a straight acceleration over a distance λ_i.

‡ The cross section for such energy-absorbing collisions is more than 20 times smaller than the elastic cross section below 9 eV, but is only about 3 times smaller in the range 9–11 eV. See for example Yee et al., figure 1.

\# The average number of collisions is simply the total distance travelled divided by λ_e. The total distance is λ_e (from the first collision) plus the integral over time of electron velocity, $\int(2U/m)^{1/2}\,dt$, from $t = (2m_e\lambda_e/eE)^{1/2}$ to $t = t_U^{rand}$ given by equation A-10. (This integral is most easily performed by changing the integration to dU by substituting for $dU/dt = \Delta U/\Delta t_{coll}$ from a previous footnote.) Inverting equation A-11 results in the well-known "random walk" in kinetic energy space U, where the energy gained is proportional to the square root of the number of elastic collisions or "steps."

$$\tau_d = \tau_d^{(0)} \cong \frac{3.7 \cdot 10^{-9}}{\sqrt{E}} \exp\left(\frac{h}{14\ \mathrm{km}}\right) \text{ seconds} \tag{A-12a}$$

For $5\ V/\lambda_e \leq E \leq 20\ V/\lambda_e$, ionization occurs after a series of elastic collisions (from 56 to 2.6 such collisions), and*

$$\tau_d = \tau_d^{\mathrm{rand}} \cong \frac{1.56 \cdot 10^3}{E^2} \exp\left(\frac{-h}{7\ \mathrm{km}}\right) - \frac{1}{3}\sqrt{\frac{2m_e\lambda_e}{eE}} \text{ seconds} \tag{A-12b}$$

The knees in the curves in figures 2 and 4 correspond to electron energies of $eE\lambda_e = 20$ eV, where the models of electron acceleration expressed by equations A-12a and A-12b are joined.

Number of Electron Doublings

We must now compute the number of electron doublings required to raise the plasma frequency and thereby reduce the skin depth to the cut-off length given by equation 1.

At any particular local electron density n, the associated "plasma frequency"[19] is

$$\omega_p \equiv \sqrt{\frac{ne^2}{\varepsilon_0 m_e}} = 56.4\sqrt{n}\ \mathrm{s}^{-1} \text{ (where } n \text{ is in m}^{-3}\text{)} \tag{A-13}$$

It is shown in appendix B that, when the plasma becomes dense enough to satisfy the condition

$$\omega_p^2 \geq \omega\bar{\nu}_c \tag{A-14}$$

where $\bar{\nu}_c$ is the collision frequency, the local skin depth δ can be approximated in terms of ω_p using the simple formula

$$\delta \cong \frac{c}{\omega_p}\sqrt{\frac{2\bar{\nu}_c}{\omega}} \tag{A-15}$$

Here ω is the characteristic frequency of the microwave pulse, c is the speed of light, and

* Note that we have set $U = 26.8$ eV in equation A-10 to obtain equation A-12b. Within the given range of E, the second term in equation A-12b is always small compared to the first.

$$\bar{\nu}_c \equiv Nv\,(\sigma_e + \sigma_i) \approx 5 \cdot 10^{12} \exp\left(\frac{-h}{7\ \mathrm{km}}\right) \mathrm{s}^{-1} \text{ for } 1\ \mathrm{eV} \le U \equiv \frac{mv^2}{2} \le 1{,}000\ \mathrm{eV} \quad \text{(A-16)}$$

is the approximate average collision frequency* for electron collisions with air molecules over the entire energy range of interest.[20]

If we require from equation 1 that $\delta \le ct_{co}$ for cut-off, and substitute δ from equation A-15, we can express the cut-off condition in terms of the plasma frequency:

$$\omega_p^2 \ge \frac{2\bar{\nu}_c}{\omega t_{co}^2} \quad \text{(A-17)}$$

When $\omega < 1/t_{co}$, according to the Heisenberg uncertainty principle the pulse that sneaks through the ionized region will be dominated by higher frequencies, making it appropriate to substitute $\omega \approx 1/t_{co}$ as a typical frequency of the pulse. Since this substitution is then conservative when $\omega > 1/t_{co}$ (leading to an overestimate of ω_p in that case and hence of the cut-off time), we will therefore use $\omega = 1/t_{co}$ in equation A-17. In this approximation, equation A-17 becomes

$$\omega_p^2 \ge \frac{2\bar{\nu}_c}{t_{co}} \text{ (subject to } \omega_p^2 \ge \omega\bar{\nu}_c\text{)} \quad \text{(A-18)}$$

To obtain an expression for the cut-off time itself, we must express the exponentially increasing ω_p in terms of t and solve equation A-18 for $t = t_{co}$. Using equation A-13 for ω_p and equation A-2 for n, we obtain

$$\omega_p^2 = 56.4^2\, n_a\, 2^{t/\tau_d} \quad \text{(A-19)}$$

Using equation A-18 with this expression and taking the natural log results in

$$t_{co} = \frac{\tau_d}{\ln(2)} \ln\left(\frac{\max\left[\frac{2\bar{\nu}_c}{t_{co}}\ ;\ (\omega\bar{\nu}_c)\right]}{56.4^2 n_a}\right) \quad \text{(A-20)}$$

where the *max* function has been used for notational convenience in order to incorporate both conditions from equation A-18. We then use equations A-12a and A-12b to

* $\bar{\nu}_c$ is, of course, dominated by elastic collisions below 10 eV and by ionizing collisions above 40 eV. Note that, although the approximation for the collision frequency $\bar{\nu}_c$ in equation A-16 is different from the more exact approximation used previously of constant σ_i and σ_e within certain limited energy ranges, the approximation here is adequate for the purposes of equation A-15 and is therefore chosen as a matter of computational convenience.

express the doubling time τ_d in terms of E and h, equation A-16 for the collision frequency $\bar{\nu}_c$, and equation A-1 to express the ambient electron density $n_a(h)$, and solve equation A-20 for t_{co} as a function of E and h.*

Dependence on Microwave Frequency

Equation A-20 holds in the low-frequency approximation $\omega\tau_d < 1$ and for electric fields satisfying the conditions in equations A-12a and A-12b. For electric fields outside this range (i.e., $E < 5$ V/λ_e and $E > 1{,}000$ V/λ_i), breakdown is inhibited by nonionizing inelastic collisions and by the decreasing ionization cross section, respectively, and t_{co} would be correspondingly greater than the value that would be predicted by our formulas. For very low fields ($E\lambda_e << 5$ volts), t_{co} would increase dramatically and the cut-off process would eventually be squelched completely.

Values of t_{co} that satisfy equation A-20, along with the domain of validity of the model's assumptions, are shown in figure 2, as a function of altitude and the electric field strength of the pulse, for a microwave frequency of 0.1 gigahertz. At 3 gigahertz, the low-frequency approximation causes the $\omega\tau_d < 1$ boundary to move left by a factor of 30 along the t_{co}-axis and the $\omega/\bar{\nu}_c < 1$ boundary to move down by $7 \cdot \ln(30) \cong 24$ kilometers. Cut-off times were computed for this frequency as well and, where different, were found to be only about a line's width larger at each electric field than those plotted for 0.1 gigahertz. (This is due to the relative insensitivity of the model to frequency as long as the low-frequency approximation is satisfied and to the natural-log factor in equation A-20.) Thus, the curves on figure 2 for $t_{co} \leq 3$ nanoseconds ($E \geq 1$ MV m^{-1}) are also justified and very nearly correct for 3-gigahertz microwave frequencies as well.

APPENDIX B: DERIVATION OF THE ELECTROMAGNETIC "SKIN DEPTH"

In order to understand the features of intense electromagnetic wave propagation through an ionizing gas or plasma, we employ a one-dimensional plasma-fluid model combined with Maxwell's equations to describe plane-wave propagation in the z-direction:

$$\nabla_z \times \mathbf{E} = \frac{-\partial \mathbf{B}}{\partial t} \qquad \text{(B-1)}$$

$$\nabla_z \times \mathbf{B} = \mu_0 \mathbf{J} + c^{-2}\frac{\partial \mathbf{E}}{\partial t} \qquad \text{(B-2)}$$

$$\frac{\partial n}{\partial t} + \nabla_z \cdot (n\mathbf{u}) = \nu_i n \qquad \text{(B-3)}$$

$$\frac{\partial (n\mathbf{u})}{\partial t} = \frac{-en}{m_e}(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - n\bar{\nu}_c \mathbf{u} \qquad \text{(B-4)}$$

* When the first term in square brackets is the maximum, the solution for t_{co} is actually *implicit*. However, because of the insensitivity of the natural-log factor, a few numerical iterations quickly converge on a self-consistent value for t_{co}.

where $-e$, m_e, and n are respectively the electron charge, mass, and number density, **u** is the electron fluid velocity, $\mu_0 \equiv 1/(\varepsilon_0 c^2) = 4\pi\cdot10^{-7}$ H m^{-1} in MKSA units, $\mathbf{J} = -ne\mathbf{u}$ is the current density, $\nabla_z \equiv \hat{\mathbf{z}}\ \partial/\partial z$, ν_i is the electron ionization frequency (implicitly dependent on air density), and $\bar{\nu}_c$ is the momentum-transfer collision frequency averaged over the electron energies of interest.[21] Frequencies (although low compared to the rate of electron collisions) can be assumed to be high enough so that ions contribute only as an immobile charge-neutralizing background. The energy equation—which would include the $\mathbf{u}\cdot\mathbf{E}$ term as well as terms for the air-molecule-exciting *inelastic* collisions and the energy transfer associated with the ionization rate ν_i—is omitted here, because it decouples from the transverse waves in the linearized approximation to follow.* Without loss of generality, we may also assume that the electric and magnetic fields of the *transverse* waves are polarized in the $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ directions, respectively.

We now linearize equations B-1 to B-4, ignoring products of the first-order quantities E_x, E_z, B_y, u_x, u_z, and n, while taking the zeroth-order electron density from equation B-3 to be:

$$n_o(t) = n_a \exp(\nu_i t)^{\dagger} \qquad \text{(B-5)}$$

where n_a is the ambient ("seed") electron density. Assuming plane waves with frequency ω propagating in the $\hat{\mathbf{z}}$ -direction, the space and time variation can be represented by $\exp(i[kz - \omega t])$, where i is the square root of -1. Equations B-1 to B-4 then become

$$ikE_x = i\omega B_y$$

$$-ikB_y = -e\mu_0 n_0 u_x - (i\omega/c^2)E_x$$

$$0 = -e\mu_0 n_0 u_z - (i\omega/c^2)E_z$$

$$-i\omega n + ikn_0 u_z = \nu_i n$$

$$-i\omega n_0 u_x = (-en_0/m)E_x - n_0\bar{\nu}_c u_x$$

$$-i\omega n_0 u_z = (-en_0/m)E_z - n_0\bar{\nu}_c u_z$$

in which the B_y, E_x, and u_x components (in the first, second, and fifth of these six equations) decouple from the others to give the dispersion relation $k(\omega)$ for the transverse (electromagnetic) waves. This can be written in the form of a complex dielectric function $\varepsilon(\omega)$ as:

* See, for example, Yee et al., pp.1238–1240. The pressure term, $\nabla_z(nU)$ is also omitted from the momentum equation, equation B-4, for the same reason; it affects only the longitudinal plasma waves.

† Comparing this with the notation in equation A-2, we see that the ionization frequency is simply $\nu_i = \ln(2)/\tau_d$, where τ_d is the "doubling time" in our model.

$$\varepsilon(\omega) \equiv \left(\frac{k(\omega)c}{\omega}\right)^2 = 1 - \frac{\omega_p^2}{\omega^2 + i\bar{\nu}_c\omega} \tag{B-6}$$

The imaginary part of the wavenumber k incorporates the loss of energy due to electron collisions with air molecules, causing the electromagnetic fields of the wave to decay exponentially in a characteristic plasma length δ, called the "skin depth":

$$\delta \equiv \frac{1}{\Im(k)} = \frac{c}{\Im\sqrt{\varepsilon(\omega)}} \tag{B-7}$$

The wave's power, which is proportional to the square of the field strength, thus decays by a factor of $e^2 \approx 7$ in each skin depth along the direction of propagation.

Using the relation

$$\Im(\sqrt{a+ib}) \equiv \sqrt{\frac{\sqrt{a^2+b^2}-a}{2}}$$

along with the low-frequency approximations $\bar{\nu}_c >> \omega$ and $\omega_p^2 >> \omega\bar{\nu}_c$, we obtain*

$$\delta \cong \frac{c}{\omega_p}\sqrt{\frac{2\bar{\nu}_c}{\omega}} \tag{B-8}$$

Since ω_p grows exponentially with time, other multiplicative factors involving ω and $\bar{\nu}_c$ that come out of the exact expression for δ have only an extremely small effect on the cut-off time and need not be included here. For the same reason, the condition $\omega_p^2 >> \omega\bar{\nu}_c$ is guaranteed to occur, if not before, then almost simultaneously with the cut-off time. (See discussion at equation A-18.)

* These approximations also satisfy the definition of a good conductor, $\Im(\varepsilon[\omega]) >> |\Re(\varepsilon[\omega])|$. Alternatively, but with less insight into the plasma dynamics, equation B-8 can be derived from equation 7.49 (with the electron binding frequency set to zero), and equations 7.58, 7.68, and 7.77 of J.D. Jackson, *Classical Electrodynamics*, (New York: Wiley & Sons, 1975), pp.284–298.

Note that since there cannot be fewer overall collisions than ionizing collisions ($\bar{\nu}_c \geq 1/\tau_d$), the low-frequency condition $1/\tau_d \geq \omega$ also implies that $\bar{\nu}_c \geq \omega$. If we were to examine the other regime—the collisionless regime where $\bar{\nu}_c/\omega << 1$, then the plasma skin depth would simply be $\delta \cong c/\omega_p$, under the conditions $\omega_p^2 > \omega^2$ and $\Re(\varepsilon) << -1$. However, in this high-frequency regime our model is not appropriate.

NOTES AND REFERENCES

1. Jick H. Yee, R.A. Alvarez, D.J. Mayhall, D.P. Byrne, and J. DeGroot, "Theory of Intense Electromagnetic Pulse Propagation Through the Atmosphere," *Physics of Fluids* **29**, 4 (April 1986), pp.1238–1244; presents theory, numerical simulations, and experimental data.

2. See for example Theodore B. Taylor, "Third Generation Nuclear Weapons," *Scientific American* **256**, 4 (April 1987), p.30; Michael D. Lemonick, "A Third Generation of Nukes," *Time*, 25 May 1987, p.36; and Dan L. Fenstermacher, "The Effects of Nuclear Test Ban Regimes on Third-generation-weapon Innovation," *Science & Global Security* **1**, 3–4 (Spring 1990), pp.209–220.

3. For a detailed discussion of the technical issues involved in a test ban on nuclear weapons, see *Toward a Comprehensive Nuclear Warhead Test Ban* (Washington DC and Moscow: The International Foundation, January 1991).

4. The one-dimensional model for electrically stressed semiconductor junctions first proposed by Wunsch predicts that, for pulses shorter than about 100 nanoseconds, junction failure in integrated circuits will occur for a certain amount of *energy* in the pulse, not power, because the heat cannot be dissipated in such a short time. (D.C. Wunsch and R.R. Bell, "Determination of Failure Levels of Semiconductor Diodes and Transistors Due to Pulse Voltages," *IEEE Trans. Nuclear Science*, NS-15, 6 [December 1968], pp.244–259; see also Robert J. Antinone, "How to Prevent Circuit Zapping," *IEEE Spectrum*, April 1987, p.37.)

5. It is interesting to compare these damage-threshold *energies* with experimental results of *continuous-wave* illumination of integrated circuits. Tests have shown that one or two out of three unhardened commercially-available microprocessor boards were upset by 2.5 or 25 W m^{-2} of continuous 1-gigahertz microwaves, respectively. (W.W. Everett III, and W.W. Everett, Jr., "Microprocessor Susceptibility to RF Signals—Experimental Results," in *IEEE Southeastcon '84 Conference Proceedings*, IEEE Catalog No. 84CH1984-4 [1984], pp.512–516.) If we assume thermal equilibrium is reached in about 1 millisecond, as would be suggested by the Wunsch model, then this would roughly correspond to an absorbed energy *from a single short pulse* (with 0.01 m^2 absorption cross section, see below) of 25–250 microjoules—which does, in fact, fall in the middle of the range of estimates given for integrated circuits (see table 1).

6. $\lambda_p^2/4\pi$ is the *maximum* absorption cross section of a monopole (omnidirectional) antenna. For antenna-like components whose characteristic dimensions are much less than λ_p, however, the inevitable mismatch to the component's internal impedance will cause a substantial portion of the incoming energy fluence to be reflected; thus the *effective* absorption cross section for absorbers much smaller than a wavelength would be considerably smaller than that given by this formula. [E.C. Jordan and K.G. Balmain, *Electromagnetic Waves and Radiating Systems,* 2nd Edition, (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968], p.377.) Similarly, a *directional* antenna will collect incoherent power only over an area on the order of one wavelength squared, and such a signal will not propagate down a solid lead for many wavelengths. (Compare Samuel Glasstone and Philip J. Dolan, *The Effects of Nuclear Weapons*, 3rd edition, [Washington DC: US Government Printing Office, 1977], p.524.)

7. See Conrad L. Longmire, "On the Electromagnetic Pulse Produced by Nuclear Explosions," *IEEE Transactions on Antennas and Propagation* **AP-26**, 1 (January 1978), p.4.

8. For instance, the effective area of a circular aperture for transmitting electromag-

netic energy is about $(2D/\lambda_p)^4$ times its actual area when its diameter $D \leq \lambda_p$, i.e., 10^{-4} when $D = \lambda_p/20$. (J.E. Casper, et al., "Performance of Standard Aperture Shielding Techniques at Microwave Frequencies," in *IEEE 1988 International Symposium on Electromagnetic Compatibility*, IEEE Catalog No. 88CH2623-7 [1988], pp.218–222.) A 3 × 0.3–centimeter slit in a 100 × 20–centimeter cylinder provides more than a factor of 100 shielding (for example, to an internal 22-centimeter wire) even at the slit's resonant frequency of 4 gigahertz—where the length of the slit is roughly a half-wavelength—and 10^3 to 10^5 shielding at both higher and lower frequencies. (R.J. King, et al., "Phenomenology of Electromagnetic Coupling—Part II," Technical Report NTIS DE86000838, Lawrence Livermore National Laboratory Report UCID20215 part II, 1985.)

9. Such diodes are often used in combination with gas-filled waveguides (plasma limiters) which, as would be expected, have turn-on times of about 10 nanoseconds, as well as ferrite devices to absorb large surges. (See H. Keith Florig, "High Power Microwave Coupling and Effects on Electronics," *Annales de Physique, Colloque no. 2* **14**, 6, supplement, (December 1989), p.114, and references therein.]

10. We are grateful to Stanislav Rodionov of the Soviet Space Research Institute for bringing this possibility to our attention.

11. Microwave directed-energy weapons of the latter type might, for example, be based on the principles of virtual-cathode oscillators ("vircators") or magnetic-flux compression. (See Fenstermacher, "The Effects of Nuclear Test Ban Regimes on Third-generation-weapon Innovation.") In the vircator-like concept, gamma rays from a nuclear detonation would generate an electron beam within the device sufficiently intense so as to create a self-repelling virtual cathode that would oscillate—depending on its electron density and plasma frequency—at up to tens of gigahertz. Such a device might be able to generate a several-nanosecond pulse with a directivity of 0.1 to 0.01 radians or narrower. (Diffraction spreads electromagnetic beams to angular widths of at least $1.2\lambda_p/D$, where D is the diameter of the focusing optics and λ_p is the pulse wavelength.)

A second possibility involves magnetic flux compression between a cylindrical conducting armature containing a nuclear explosive and a rigid current-carrying coil that would surround it. A directed EMP might be generated from the resulting current pulse using some kind of antenna. Here, pulse lengths might be made short enough to lie in the gigahertz range, with directivities for 3-meter antennas, say, of around 0.1 radian. For discussions of (conventional-explosive-powered) magnetic-flux compression devices, see A.D. Sakharov, "Magneto-implosive Generators," *Soviet Physics* (*Uspekhi*) **9**, 2 (September–October 1966), pp. 294–299; and E.C. Cnare, R.J. Kaye, and M. Cowan, "A 2 MJ Staged Explosive Generator," in M.F. Rose and T.H. Martin, eds., *Proceedings of the 4th IEEE Pulse Power Conference*, Albuquerque, New Mexico, 6–8 June 1983, pp.102–104.

Although it will not be discussed further here, an additional problem with any device attempting to focus its own microwave beam is electron emission at its metal surfaces or antenna, which occurs at field strengths of around 30–100 MV m^{-1} and can short-circuit a beam.

12. See Fenstermacher, "The Effects of Nuclear Test Ban Regimes on Third-generation-weapon Innovation," for the basis for these assumptions. Recall that 1 kiloton ≡ $4.2 \cdot 10^{12}$ joules. At higher frequencies, attenuation due to resonant absorption by molecules in the atmosphere will reduce the maximum fluence. For example, water vapor and oxygen have resonances at 22 and 60 gigahertz, which cause attenuations in the lower atmosphere of 0.1 and 10 dB km^{-1}, respectively, within several gigahertz of these frequencies. See Florig, "High Power Microwave Coupling and Effects on Electronics."

13. *Conventionally powered* microwave generators such as gyrotrons, magnetrons, vircators, and explosive flux-generators continue to be of great interest to the military, but applications are generally short-range and quite distinct from the potential strategic missions of nuclear-powered counterparts. See, for example, V.L. Granatstein, "High Power and High Peak Power Gyrotrons: Present and Future Prospects," *Intl. Journal of Electronics* 57, 6 (June 1986), pp.787–799; and M.D. Pocha, "High-Power Microwave Generation," *Energy and Technology Review*, July–August 1990, p.72.

14. For the value of n_a near the earth's surface, see O.H. Gish, "Atmospheric Electricity," in J.A. Fleming, ed., *Physics of the Earth, Vol. 8: Terrestrial Magnetism and Electricity*, (McGraw-Hill, 1939), pp.166 ff. (Gish assumes an ionization rate from cosmic rays and other natural radiation of 10^7 m^{-3} s^{-1} and an electron lifetime of 10^{-5} seconds.) Above 36 kilometers, the daytime and nighttime profiles of the "D-region" (approximately 60–90 kilometers altitude) can be modeled more exactly in equation A-1's power law by using 53 and 68 kilometers, respectively, in place of 60 kilometers. Compare J.K. Hargreaves, *The Upper Atmosphere and Solar-Terrestrial Relations* (New York: Van Nostrand, 1979), p.60 (figure 4.5).

15. Hargreaves, pp.53–54. Up to an altitude of 100 kilometers, the exponential scale-height varies with altitude by only about ±20 percent, primarily due to temperature variations, and is given by $k_B T/mg$, where $k_B \cong 1.38 \cdot 10^{-23}$ J K^{-1} is Boltzmann's constant, and $g \cong 9.8$ m s^{-2} is the acceleration at sea level due to gravity. Up to 100 kilometers, the average mass of air molecules is $m \cong 29$ atomic mass units $\cong 4.8 \cdot 10^{-26}$ kilograms, and the average temperature is about 240 K (–28 °F). This gives a scale-height of about 7 kilometers.

16. See for example Yee et al., figures 1 and 2.

17. Ibid.

18. See for example Gerhard Herzberg, *Molecular Spectra and Molecular Structure I: Spectra of Diatomic Molecules*, second edition, (New York: Van Nostrand, 1950). pp.444–450; D.C. Cartwright, W.J. Hunt, W. Williams, S. Trajmar, and W.A. Goddard III, "Theoretical and Experimental (Electron-Impact) Studies of the Low-Lying Rydberg States in O_2," *Physical Review A*, 8 (1973), pp.2436–2448.

19. See for example J.D. Jackson, *Classical Electrodynamics, 2nd edition* (New York: John Wiley & Sons, 1975), p.321.

20. Derived from Yee et al., figures 1 and 2, given the density of air molecules in the atmosphere. The value $\bar{v}_c/p \cong 5 \cdot 10^{12}$ s^{-1} atm^{-1} over a wide range of E/p (p is the atmospheric pressure) can also be derived from Sanford Brown, *Basic Data of Plasma Physics, 2nd Edition* (Cambridge, Massachusetts: MIT Press, 1966), p.84 (figure 4.9).

21. Yee, et al., p.1239. See especially equation 14 of that reference.