Science & Global Security, 1990, Volume 2, pp.79–86
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Printed in the United States of America

Appendix
LASER BEAM SCATTERING IN THE ATMOSPHERE

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Most suggested schemes for verification of the power of ASAT lasers are based on measurements of laser light scattered in the atmosphere. The feasibility of these measurements is determined by two physical factors:

- The intensity of the scattered light, which depends on the power of the laser, its wavelength, the geometry of the beam and detector, and the physical properties of the scattering medium (i.e. the atmosphere)
- The brightness of the atmospheric background light at the wavelength being observed, which depends on the atmospheric conditions and the position of the sun.

These factors essentially determine the signal-to-noise characteristics of the data taken by a verification monitor. In this note we present a brief collection of formulas and tables that can be used for estimating the levels of scattered light and give a general formula for the background light from the atmosphere.

There are two main mechanisms by which a laser beam propagating through the atmosphere is scattered: Rayleigh scattering by individual molecules and scattering by aerosols (small particles suspended in the atmosphere). We assume that the power per unit area of the laser beam is low enough that it does not change the optical characteristics of the atmosphere through which it propagates (i.e. we assume linear losses). The intensity of a beam propagating through a medium with linear losses is given by $I(x) = I_0\exp(-\beta x)$, where I_0 is the intensity of the beam as it enters the medium (at $x = 0$), $I(x)$ is the beam intensity a distance x into the medium, and β is the "extinction (or attenuation) coefficient" for the various scattering (and absorption) processes. (An upward propagating beam will, of course, experience variable attenuation with altitude; see below.)

The extinction coefficient for Rayleigh scattering is inversely proportional to the fourth power of the wavelength and is given approximately by[1]

$$\beta_R(\lambda) \approx \frac{32\pi^3(n - 1)^2}{3N\lambda^4} \tag{1}$$

where N is the number density of molecules ($3{\cdot}10^{19}$ molecules cm^{-3} at sea level), λ is the wavelength of the radiation, and n is the refractive index (which depends on the wavelength). The dependence of the extinction coefficient for atmospheric Rayleigh scattering on wavelength and altitude is shown in table 1. It varies with altitude as the atmospheric density.

Table 1: Extinction coefficients for molecular scattering versus wavelength at sea level

λ μm	β_R km^{-1}	λ μm	β_R km^{-1}
0.30	$1.4 \cdot 10^{-1}$	0.65	$5.9 \cdot 10^{-3}$
0.32	$1.1 \cdot 10^{-1}$	0.70	$4.0 \cdot 10^{-3}$
0.34	$8.5 \cdot 10^{-2}$	0.80	$2.5 \cdot 10^{-3}$
0.36	$6.7 \cdot 10^{-2}$	0.90	$1.6 \cdot 10^{-3}$
0.38	$5.2 \cdot 10^{-2}$	1.06	$8.5 \cdot 10^{-4}$
0.40	$4.3 \cdot 10^{-2}$	1.26	$4.1 \cdot 10^{-4}$
0.45	$2.6 \cdot 10^{-2}$	1.67	$1.3 \cdot 10^{-4}$
0.50	$1.7 \cdot 10^{-2}$	2.17	$4.6 \cdot 10^{-5}$
0.55	$1.2 \cdot 10^{-2}$	3.50	$6.8 \cdot 10^{-6}$
0.60	$8.2 \cdot 10^{-3}$	4.00	$4.0 \cdot 10^{-6}$

Table 2: Physical parameters of different types of aerosol (hazes, precipitation, and clouds)

	N cm^{-3}	c	a_m μm	α	γ
Marine and coast	100	$5.3 \cdot 10^{4}$	0.05	1	0.5
Continental	100	$5.0 \cdot 10^{6}$	0.07	2	0.5
High-altitude and stratospheric	100	$4.0 \cdot 10^{5}$	0.10	2	1
Drizzle and light rain	10^{-4}	$5.3 \cdot 10^{5}$	$<5 \cdot 10^{2}$	1	0.5
Heavy rain	10^{-3}	$5.0 \cdot 10^{7}$	$>5 \cdot 10^{2}$	2	0.5
Hail with significant content of small particles	10^{-5}	$4.0 \cdot 10^{-4}$	$1 \cdot 10^{4}$	2	1
Cumulus and stratus clouds, mists	100	2.4	4.00	6	1
Clouds with corona (iridescence)	100	$1.1 \cdot 10^{-2}$	4.00	8	3
Nacreous (mother of pearl) clouds	100	5.6	2.00	8	3

Source: D. Daimengan, *The dispersion of electromagnetic waves by spherical polydispersion particles*, (Moscow: Mir, 1971).

The (normalized) angular distribution of Rayleigh scattered radiation is given by a standard dipole radiation pattern[2]

$$f(\theta) = \frac{3(1 + \cos^2\theta)}{4} \tag{2}$$

Here θ is the scattering angle, the angle between the propagation direction of the laser beam and the direction from the scattering volume to the detector.

Because the efficiency of Rayleigh scattering decreases sharply with wavelength. Rayleigh scattering predominates over aerosol scattering (see below) only in the near-ultraviolet and blue parts of the spectrum.

Aerosol scattering becomes important at longer wavelengths[3] and has a much more complicated angular distribution. The intensity and angular distribution of aerosol scattering depend strongly on the local aerosol composition of the atmosphere, which varies widely with climate, precipitation, etc. Therefore no simple formulas are available for these quantities, and the scattering process must be modeled through detailed calculations.

Modeling the aerosol component of the atmosphere involves three steps:

- Description of the physical characteristics of the individual aerosol particles
- Calculation of the optical characteristics based on the specified physical characteristics
- Determination of the spatial distribution of the aerosol particle density.

A modified normalized gamma distribution

$$g(a) = ca^{\alpha}\exp(-ba^{\gamma}) \tag{3}$$

is commonly used to fit the aerosol size distribution.[4] Here a denotes the particle radius and c, α, b, and γ are empirical parameters. b is related to the most probable radius a_m by the formula $b = \alpha/(\gamma a_m^{\gamma})$. The fitted parameters of the size distributions of different types of aerosol are given in table 2.

The optical parameters of an aerosol are calculated from the size distribution of the particles and their refractive index and geometric characteristics. The results of such theoretical calculations must be corrected on the basis of experimental data taken at a given monitoring site. Table 4 shows the approximate dependence of the aerosol extinction coefficient and the angular distribution of the scattered light for an optical model of cumulus and stratus clouds. Both the extinction coefficient and the angular distribution depend on the laser wavelength and the type of aerosol. The extinction coefficient for aerosol scattering can be approximated empirically as follows:[5]

$$\beta_a(\lambda) = \beta_a(0.59 \text{ microns})(k_0 + k_1\lambda^{-k_2}) \tag{4}$$

Some empirical values of k_0, k_1, and k_2 are given in table 4 for different atmospheric conditions. The atmospheric conditions in table 4 are, of course, imprecisely defined and are also not very relevant to conditions under which lasers would be used; but they do illustrate the great variability of the atmosphere with respect to aerosol scattering.

The height distribution of atmospheric aerosols obeys an empirical formula given by Elterman.[6] At altitudes below 5 kilometers, the aerosol extinction coefficient varies approximately as

$$\beta_a(z,\lambda) = \beta_a(0,\lambda)\exp\left(\frac{-z}{z_a}\right) \tag{5}$$

where z_a is determined by matching the formula to the observed value at $z = 5$ kilometers. Above 5 kilometers, the aerosol extinction coefficient is relatively independent of the value at sea level (Elterman's values are given in table 4).

Table 3: Optical model of a cloud with microstructure parameters $a_m = 4\mu m$, $\alpha = 6$, $b = 1.5$, $\gamma = 1$, $N = 100\ cm^{-3}$

λ	β				$f(\theta)/2\pi$			
μm	km^{-1}	θ: 0	30	60	90	120	150	180
0.45	16	620	0.36	0.045	0.0054	0.0073	0.026	0.10
0.70	17	270	0.26	0.033	0.0054	0.0072	0.26	0.10
1.2	17	97	0.36	0.050	0.0092	0.0081	0.030	0.091
1.5	18	68	0.35	0.053	0.011	0.0092	0.032	0.11
1.6	18	54	0.35	0.055	0.012	0.010	0.034	0.12
1.9	18	41	0.34	0.057	0.013	0.011	0.030	0.089
2.3	18	29	0.38	0.058	0.014	0.015	0.030	0.091
3.0	18	32	0.13	0.018	0.0077	0.0059	0.0056	0.0056
3.9	21	11	0.42	0.073	0.021	0.015	0.030	0.038
5.3	24	7.6	0.41	0.049	0.014	0.0091	0.013	0.0085
6.1	20	8.3	0.34	0.032	0.0082	0.0042	0.0035	0.0030
8.2	19	4.8	0.55	0.044	0.011	0.0054	0.0052	0.0055
10	11	3.7	0.68	0.045	0.0093	0.0045	0.0039	0.0047
12	10	2.9	0.75	0.058	0.011	0.0059	0.0051	0.0052
17	17	1.6	0.72	0.130	0.033	0.017	0.014	0.014

Background Radiation from the Atmosphere

The background light from the atmosphere has several components:

- molecular scattering of sunlight
- aerosol scattering of sunlight
- thermal (black body) radiation of the atmosphere
- nonthermal radiation from the atmosphere.

The relative contributions of these components depend on the wavelength, the position of the sun, and the characteristics of the atmosphere (cloud cover, etc.). In the absence of clouds, the following formula can be used to estimate the background in the visible and near infrared:[7]

$$B_H = \left[\pi S_\lambda \frac{2.060\, \tau_R (1 + \cos^2\phi) + \eta(\exp[-3\phi] - 0.009)}{\tau_a + \tau_R} \right.$$
$$\times \left(\frac{\exp[-\tau_a \sec\xi_0] + \exp[-\tau_a \sec\xi]}{\sec\xi - \sec\xi_0} \sec\xi + \frac{c_H S_\lambda \exp[-\tau_a \sec\xi_0]\, \tau_a \sec\xi_0}{4} \right. \tag{6}$$
$$\left. \left. \times \left(\frac{1 - \exp[-\tau_a \sec\xi]}{1 - \exp[-\tau_a \sec\xi_0]} \right) \right) \right] \exp[-\tau_0 \sec\xi_0]$$

Here

πS_λ is the solar constant as a function of wavelength,

τ_R, τ_a, and τ_0, which are all functions of wavelength, are the optical depths for attenuation of light by Rayleigh and aerosol scattering and molecular (by ozone or other atmospheric molecules) absorption respectively,

$c_H = 0.133 + 1.33q$, where q is the albedo of the underlying surface,

$\eta = 2.326(\tau_a + 0.062\tau_R)$,

ϕ is the angle between the observation direction and the sun,

ξ_0 is the angle of the sun from the zenith, and

ξ is the angle of the observation direction from the zenith.

Table 4: Empirical parameters of aerosol extinction coefficients

Season	Weather type	k_0	k_1	k_2
Winter	"Ice" haze	0.248	0.447	1.24
	Winter haze	0.	0.580	1.24
	Haze with snow	0.77	0.145	1.24
Spring/ Fall	Stable spring/ fall hazes	0.04	0.585	1.02
	Mist haze	0.116	0.690	0.56
	Haze with drizzle	0.275	0.455	1.09
	With drizzling rain	0.605	0.215	1.34
Summer	Radiation fog *after heavy rain*	0.	0.400	1.88
	Stable haze with visibility ≥ 4 kilometers	0.06	0.360	1.88

Table 5: Dependence of aerosol extinction coefficient β (in km^{-1}) on altitude at altitudes higher than 5 kilometers

Z *km*	**Wavelength** *microns* 0.40	0.55	0.70	1.06	1.26	1.67	2.17
5	$6.4\cdot10^{-3}$	$5.0\cdot10^{-3}$	$4.3\cdot10^{-3}$	$3.6\cdot10^{-3}$	$3.4\cdot10^{-3}$	$3.1\cdot10^{-3}$	$2.7\cdot10^{-3}$
10	$4.0\cdot10^{-3}$	$3.2\cdot10^{-3}$	$2.7\cdot10^{-3}$	$2.2\cdot10^{-3}$	$2.2\cdot10^{-3}$	$2.0\cdot10^{-3}$	$1.7\cdot10^{-3}$
15	$3.4\cdot10^{-3}$	$2.7\cdot10^{-3}$	$2.3\cdot10^{-3}$	$1.9\cdot10^{-3}$	$1.8\cdot10^{-3}$	$1.6\cdot10^{-3}$	$1.4\cdot10^{-3}$
20	$1.9\cdot10^{-3}$	$1.5\cdot10^{-3}$	$1.3\cdot10^{-3}$	$1.1\cdot10^{-3}$	$1.0\cdot10^{-3}$	$9.2\cdot10^{-4}$	$8.0\cdot10^{-4}$
25	$5.3\cdot10^{-4}$	$4.2\cdot10^{-4}$	$3.6\cdot10^{-4}$	$3.0\cdot10^{-4}$	$2.8\cdot10^{-4}$	$2.6\cdot10^{-4}$	$2.2\cdot10^{-4}$
35	$4.2\cdot10^{-5}$	$3.3\cdot10^{-5}$	$2.8\cdot10^{-5}$	$2.4\cdot10^{-5}$	$2.3\cdot10^{-5}$	$2.0\cdot10^{-5}$	$1.8\cdot10^{-5}$
50	$7.6\cdot10^{-7}$	$6.0\cdot10^{-7}$	$5.1\cdot10^{-7}$	$4.3\cdot10^{-7}$	$4.1\cdot10^{-7}$	$3.7\cdot10^{-7}$	$3.2\cdot10^{-7}$

At wavelengths above 3–4 microns, the contribution of the thermal radiation from the atmosphere becomes considerable.[8] Equation 6 is valid for a cloudless sky. Clouds cause changes in the brightness of the background and generate small-scale structure in the background.

Scattered Laser Radiation

In general, the relative brightness of a scattered laser beam can be calculated using the lidar formula and the scattered and background levels described above. The significance of the formulas given above is illustrated by the following formula for the radiance of the light scattered from a laser beam:

$$B(\theta) \propto \frac{P\,[\beta_R f_R(\theta) + \beta_a f_a(\theta)]}{d} \tag{7}$$

where P is the laser power, $\beta_R f_R$ and $\beta_a f_a$ are the products of the scattering (or extinction) coefficients and angular distributions of the scattered light for the Rayleigh and aerosol scattering respectively and d is the beam diameter.

For a laser with the parameters of the MIRACL laser (P = 1 megawatt, d = 3 meters, λ = 4 microns),[9] the typical radiance of the scattered radiation (for $\beta_R + \beta_a = 10^{-2}\ \text{km}^{-1}$ and $f(\theta) = 10^{-2}$ is

$$B = \frac{10^6 \cdot 10^{-7} \cdot 10^{-2}}{3 \cdot 10^2} = 3 \cdot 10^{-6}\ \text{W cm}^{-2}\ \text{sr}^{-1} \tag{8}$$

This value is comparable to the background for a cloudless sky in this part of the spectrum.

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