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# Assessing the Lethality of Conventional Weapons against Strategic Missile Silos in the United States, Russia, and China

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**ABSTRACT**

This paper provides a framework for assessing the vulnerability of strategic missile silos in the United States, Russia, and China to conventional weapons with any accuracy or explosive yield. Comparisons between ground motions induced by nuclear surface bursts and earth-penetrating conventional explosions were made to calculate the maximum distance at which a silo-based missile would be vulnerable to a conventional detonation. Single-shot kill probabilities then confirmed that U.S. long-range air- and sea-based precision conventional cruise missiles possess lethalities against missile silos comparable to U.S. nuclear ballistic missiles: typically well above 90%. This result suggests that long-range conventional weapons may not only be substituted for the silo counterforce targeting roles of nuclear weapons, but may have broader strategic stability and defense implications due to the relative survivability of and reliance on specific nuclear forces among nuclear powers and regional defense dynamics driving the acquisition of similar weapons by more countries.

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## Introduction

The Cold War–era debate in the United States about intercontinental ballistic missile (ICBM) silo vulnerability was mostly resolved by doing nothing. The issue arose in the 1970s and set off years of research, development, and discussion over concerns about strategic stability, but mutual vulnerability was eventually accepted, with the Soviet Union's silos in roughly as vulnerable a position if not more so from increasingly accurate U.S. ballistic missiles as U.S. silos were from improving Soviet ones. Because this vulnerability, whatever its merits or exaggerations, did not mean the vulnerability of the entire U.S. arsenal, and given the uncertainties and challenges with redressing it, nothing significant was done.

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Regarding the counterforce potential of nuclear-armed ballistic missiles today, the situation remains mostly unchanged, even as missile accuracy improvements have continued to adversely affect survivability.[1]

But the question of silo vulnerability is now intersecting with an evolving military context involving increasingly accurate long-range conventional weapons that lacks a technical basis for analysis.[2] While no U.S. government report or statement has ever specifically mentioned U.S. conventional capabilities against strategic missile silos,[3] these capabilities have been a long-standing concern among many officials in Russia.[4] And since 2011, following the ratification of the New Strategic Arms Reduction Treaty (New START), Russia has consistently stated that U.S. long-range precision-guided conventional weapons be included in any future nuclear arms control agreement, along with U.S. missile defense and other space capabilities.[5] These weapons currently and should foreseeably be expected to determine Russian nuclear force deployments.

Russia also has these precision capabilities (expanded since 2010 and first used in Syria in 2015),[6] but these stocks have been depleted with their use in Ukraine following Russia's 2022 invasion, and it remains unclear how quickly they may be replenished. It is also unclear how capable Russian weapons perform compared to U.S. weapons (i.e., accuracies, penetration of air defenses, explosive yields, etc…). Yet rebuilding and fielding a significant long-range precision conventional strike capability should be expected to be among Russia's top priorities given its by now decades-long awareness of the limits and perils of relying on nuclear weapons against similar U.S. conventional weapons.

Analyses of China's precision conventional capabilities usually focus on ballistic missiles capable of reaching targets in Asia, typically those that would be relevant in a conflict over Taiwan, including U.S. naval assets. They do have long-range conventional cruise missiles, but it is not clear whether they possess them in sufficient numbers or have the performance capabilities to pose a threat to strategic missile silos in Russia or the United States.[7] This could change depending on the evolution of China's military. Of further relevance is that China is constructing new missile silos at three locations deeper inland, with size dimensions relevant to survivability appearing to be similar to those in Russia.[8]

While the analysis offered here could apply to any conventional weapon of any accuracy or explosive yield, explicit lethality calculations were made using only two U.S. cruise missiles with earth-penetrating capabilities: the sea-based Tomahawk land attack missile and the air-based Joint Air-to-Surface Standoff Missile (JASSM).[9] These missiles have publicly revealed accuracies and yields,[10] with ranges capable of reaching strategic missile silos in Russia or China from plausible launch locations in Europe, Asia, and the surrounding waters.[11] These U.S. weapons were also modeled against U.S. strategic missile silos as a reference for what might be possible with similar missiles from Russia, China, and perhaps inevitably many other countries.[12]

Emphasizing U.S. missiles was also motivated by the number of U.S. allies who have purchased them and now maintain them as part of their national capabilities. In addition to the United Kingdom currently possessing the Tomahawk, Australia, Canada, Japan, and the Netherlands will soon acquire it; and Poland, Finland, Australia, and Morocco currently possess the JASSM, with Germany, the Netherlands, and Japan scheduled to acquire it. Other U.S. allies have their own indigenous capabilities in this area, and those could be modeled with this analysis if they are considered relevant.[13] This evolving military context and the regional defense dynamics involved further underscore the importance of understanding these weapons.

Conventional varieties of hypersonic weapons could also be modeled here,[14] but their explosive yields would have to reach those of subsonic cruise missiles to be as lethal.[15] They may pose a threat to silos by direct impact should they become highly accurate, with an analysis by Russian experts suggesting that U.S. subsonic missiles like the Tomahawk and JASSM already able to destroy silos in this fashion.[16] Should hypersonic weapons ever become a militarily significant reality, a more detailed comparison of direct impact capabilities with subsonic missiles, possibly combined with shaped explosive charges, may prove useful, but no assessment of conventional lethality is complete unless the furthest distance at which a detonation may destroy a silo is determined. More broadly, it is far from clear that faster missiles provide more advantages or are more destabilizing than slower ones when the details of strategic conflict are considered, especially with conventional weapons.

The motivation for this analysis was to assess how long-range conventional precision weapons are shaping a key aspect of the international military context in order to provide a foundational basis for analyzing the implications. Such thinking will only become more important as countries see technology changing the strategic landscape and are preoccupied with attempts to defend themselves or become or remain preeminent as they perceive a range of dangers. Failure to analyze the capabilities and constraints of military technology in a more fundamental way will likely lead to more confusion in a world where the proliferating number of perceptions risks rendering any policy discussion dangerously incoherent.

This analysis begins with a brief history of silo vulnerability, explaining why the simple proxy model using nuclear airblast peak overpressure has limits but still remains useful. Sections that follow explain the dominant silo destruction mechanism and why it is necessary to go beyond the proxy model into the more fundamental physics of ground motion to assess the lethality of conventional weapons. An explanation of the need to consider the response of the silo's shock isolation system follows, where the lack of publicly available details here demands that a comparison be made between ground motions induced by nuclear and conventional explosions. A simple, spring model of a shock isolation system was used to uncover the key ground motion parameters needed for comparison. Ground motions induced by nuclear airblasts and earth-penetrating conventional weapons were then compared with their respective geometries as a function of depth to calculate the maximum distance at which a conventional detonation could destroy a missile inside a silo. Single-shot kill probabilities (SSKPs) for the Tomahawk and JASSM against U.S., Russian, and Chinese silos were then calculated, along with those for U.S. and Russian nuclear ballistic missiles.

The calculations of the shock isolation system appear in an Appendix. A discussion of a Chinese analysis that modeled shock isolation systems also appears there, as well as a section on the vulnerability of missile silos to direct impacts and another on missile guidance and air defense.

### *A note on units*

The sections that follow rely on sources from the 1960s and 70s about nuclear weapons tests and effects. The primary document used for conventional weapons is from the 1980s. These sources use units such as feet and inches for distance and often pounds instead of kilograms for the explosive weight of a conventional charge. This is before standard meter-kilogram-second units (commonly known at MKS units) became the adopted standard for scientific and technical work within the U.S. federal government in the early 1990s. To avoid cumbersome and confusing conversions of mathematical formulas, feet, inches, and pounds will be maintained for ground motion and explosive conventional charge weight. Meters will be used for missile accuracies (circular error probable; CEP) and lethal radii in the calculation for SSKPs (Equation 2). This will allow for consistency when referring to the primary documents cited here.

## Early modeling of silo vulnerability to nuclear weapons

The vulnerability of missile silos to nuclear explosions is typically modeled using the peak overpressure of the airblast.[17] The simple idea is that a missile silo will be destroyed once a certain threshold of overpressure is incident at its location. Past research that has provided a basis for policy discussion on this issue has started with the well-known approximation for peak overpressure ($P_o$) in pounds per square inch (psi) varying with distance from a nuclear surface burst

$$P_o = \frac{3300Y}{r^3} + \frac{192Y^{1/2}}{r^{3/2}} \text{psi} \tag{1}$$

with $Y$ the explosive yield in megatons (Mt) and $r$ the distance from the point of detonation in thousands of feet (kft).[18] While Cold War estimates of what $P_o$ was required to destroy a silo located in either the United States or the Soviet Union varied from one source to the next, all noteworthy research began from this expression, albeit occasionally translated to different units or simplified according to the circumstances.[19]

Whether the $P_o$ needed to destroy a silo is achievable for any given nuclear explosion depends on the accuracy of the warhead's delivery system and its yield $Y$. Accuracy is expressed by the CEP, the radius of the circle centered on the target within which the warhead has a 50% chance of landing. This is the standard way of expressing missile accuracy (or the accuracy of any delivery system), with 50% of warheads landing inside this radius and 50% outside it. These two factors, CEP and $Y$, then determine the lethal radius, LR, the maximum distance that a target may be from the point of an explosion and still be destroyed. If the distribution of repeated shots on a target can be assumed to be circular normal, the probability of that target being destroyed in a single shot, commonly referred to as the single-shot kill probability, SSKP, can be calculated with[20]

$$\text{SSKP} = 1 - 0.5^{\left(\frac{\text{LR}}{\text{CEP}}\right)^2} \tag{2}$$

And LRs measured in meters may be calculated from a formula derived from Equation 1[21]

$$\text{LR} = 4.54 \times 10^3 \left(\frac{Y}{H}\right)^{1/3} \left\{\sqrt{1 + \frac{2.79}{H}} + \frac{1.67}{H^{1/2}}\right\}^{2/3} \tag{3}$$

where LR replaces $r$ and $H$ representing the hardness of the target up to a certain overpressure is substituted for $P_o$. While other truncated, simpler expressions for LR can produce close results to this formula depending on a target's hardness,[22] this expression covers the entire range of target hardnesses and is straightforwardly obtained from an accepted physical basis.[23] It is not pulled from an obscure graph, the output of a classified computation, or a circular slide rule that is blindly accepted as an authoritative source.[24]

Should a higher kill probability be desired, multiple warheads could target a silo resulting in a kill probability $p(\text{kill})$ for $N$ warheads of

$$p(\text{kill})_N = 1 - (1 - \text{SSKP})^N \tag{4}$$

The importance of missile accuracy relative to explosive yield can be seen with dependence here on the parameter $NY^{2/3}/(\mathrm{CEP})^2$, after substituting the expression for LR into SSKP. This inverse square dependence explains why CEP is the dominant factor of the two when concerned with ICBM vulnerability.

These simple formulas have provided the foundation for thinking about nuclear counterforce against strategic missile silos, underpinning a core element of U.S. nuclear strategy since the 1970s.[25] They were even used by Paul Nitze, a U.S. government official throughout the Cold War and member of the delegation that negotiated the 1972 SALT I agreement, in an influential 1976 article to project the vulnerability of the United States' land-based ICBMs to a credible first strike by the Soviet Union.[26] Even though his analysis was quickly found to have excluded important factors,[27] and later that such concerns in the U.S. policy discussion were unjustified due to exaggerated Soviet missile capabilities,[28] Nitze's paper revealed how useful the models were viewed by government insiders.

## The limits of peak overpressure

It is likely a quirk of nature that the peak overpressure calculations provide a good proxy of silo vulnerability, because the underlying fundamentals involve complex details of a nuclear blast and its interaction with the ground, and that of the ground with a silo. While these details may not be important when considering nuclear weapons, they reveal part of the challenge in assessing the lethality of conventional ones.

One limitation with this proxy model is that there is no specific overpressure to which a silo is vulnerable. The Defense Intelligence Agency (DIA) revealed that targets are also sensitive to the duration of the overpressure pulse,[29] indicating some failure mode in a silo that is sensitive to the total force applied over some time (i.e., the impulse). And this duration is longer for larger yields.[30] Thus a silo could be destroyed by a 1 Mt explosion with a peak overpressure of 1,000 psi, but survive a 500 kiloton (kt) blast if the overpressure only reached the same level. The larger yield imparts a greater impulse to the silo even if located where the peak overpressure is the same for both blasts. Peak overpressure then must be based on a specific yield, or silos must be considered vulnerable to a range of peak overpressures determined by the possible yields of attacking warheads.

Another disclosed limitation was that a detonation within the LR calculated in Equation 3 could not be certain to destroy a silo, as is understood by its definition and underlying the basis for the SSKP in Equation 2.

The DIA corrected for this by developing a log-normal damage function with a continuous probability distribution to allow the target some probability of surviving if a weapon landed inside the lethal radius and some chance of being destroyed even if it did not.[31] This is understandable when soil properties can vary from one silo to the next, or the level of the soil's water saturation can vary due to the weather, level of the water table, or local climate. Such factors will impact the total force an airblast can deliver to a silo. Random variations in weapons or targets could also result in one weapon exploding with less yield than predicted or more force being required at a particular target than anticipated.

Despite this, the standard overpressure calculations are considered an appropriate method for calculating silo vulnerability. Some scholars have found these calculations to result in slightly fewer surviving missile silos (i.e., a more effective counterforce capability) than if the log-normal damage function and pulse duration are considered,[32] while others have found the two methods to be about equal.[33] Regardless, the factors underpinning the DIA calculations cannot be examined because they involve inaccessible characteristics of individual targets. The standard calculations yet remain a good approximation if the silo hardness is adjusted for the appropriate yield.

## Beginning to examine conventional lethality

What this understanding of the peak overpressure model reveals is that it cannot be applied to conventional weapons. This is first because while there are empirical models that have the overpressure of a conventional surface burst exceeding what modern-day silos may survive (a few thousand psi), the empirical data is highly uncertain close to the detonation, which must occur very close to the silo.[34] This places an extreme burden on missile accuracy, which for U.S. systems has been disclosed as less than 3 m (CEP) but cannot yet be expected to improve significantly from this level.[35] Furthermore, a conventional explosion has a much shorter blast pulse duration, ruling out the same total force being imparted to the silo. A much higher overpressure would then be required than could be confidently assumed, ruling out any technically valid basis for such a prospect. If a valid basis should ever arise to confirm the destruction of a strategic missile silo from a conventional surface explosion by the dominant destruction mechanism considered in this analysis, the issue of conventional lethality would greatly simplify. This does not appear to be the case with information currently in the public domain.

It may be possible to penetrate the cover of a missile silo, especially with shaped charges that pierce the door and spray shrapnel and molten metal into the interior of the silo after detonating.[36] But any countermeasure that significantly slows down the missile before it penetrates the cover (e.g., additional shielding, a layer of topsoil, some sort of cage or wires) may complicate this task. It would also again place a high premium on missiles being extremely accurate, likely requiring several to be launched at the silo to successfully penetrate its cover with high confidence. An argument portraying this narrowly construed attack mechanism as illusory with U.S. cruise missiles possessing 3-m accuracy was made in a Russian military journal, with the authors making the assumption that any direct impact would penetrate the armored roof of a silo with a powerful charge and decommission it.[37] The paper was also cited approvingly by an experienced Russian nuclear expert.[38] While this sort of attack might become possible with missiles accurate to within 1 or 2 m and countermeasures able to be evaded,[39] it does not currently appear to have a high chance of success. Claims of a serious conventional weapons threat to silos cannot therefore rest only on such a vulnerability.

This leaves open the question of how conventional weapons could credibly threaten missile silos given these constraints. It turns out the answer lies in the physical picture of a nuclear weapon's dominant silo destruction mechanism, which is rarely referenced in any policy discussion. With the overpressure proxy model long recognized as adequate, there was little need to consider the details involved here. But reinventing this wheel is necessary if conventional lethality is to be properly assessed.

## Moving beyond the overpressure proxy model

Missile silos are vulnerable to a variety of effects that accompany nuclear explosions: x-rays, gamma-rays, neutrons, electromagnetic pulse, thermal radiation, airblast, and airblast- and direct-induced ground motions. The magnitude of each decreases with distance from the point of detonation.[40] The ability of missile silos to withstand these effects are typically referred to as "hardness," but this should not be confused with a measurement threshold against which a silo can withstand being crushed or physically destroyed. It instead refers to an array of properties relevant for defining the survivability of the silo, missile, or supporting equipment that allows for a successful missile launch after a nuclear attack. It therefore includes many factors related to a silo's environment, as alluded to in the earlier discussion on peak overpressure.

The effect of primary concern will be the one possessing the largest lethal radius, as it will destroy a silo from a detonation point farther away than other effects. This radius is defined by the level of a silo's hardness, which is designed to withstand the airblast up to a certain threshold and all other effects—nuclear, electromagnetic, and thermal radiation—up to this level.[41] The dominant destruction mechanism has then been assumed to be either the airblast[42] or the combined effect of airblast and airblast-induced ground shock accompanying it that cause the missile inside to respond to silo displacements and velocities.[43] In 1981, the *MX Missile Basing* study from the Office of Technology Assessment went further, offering a more specific description of the dominant destruction mechanism when discussing shelter hardness:[44]

> There are several damage mechanisms to a missile from a nuclear detonation. These mechanisms are airblast, ground shock, electromagnetic pulse, radiation, and thermal effects.

Regarding airblast overpressure:

> An airblast results in overpressure destruction, and it is particularly severe on aboveground objects (such as the shelter door of a horizontal shelter) that must withstand the reflected loads of the incident shock front. For a vertical shelter, with a shelter door that is flush with the surface, there are no reflected loads, and door requirements are far less severe than for the horizontal shelter.

Continuing two paragraphs later:

> Ground motions result from the "air-slap" of the shock front hitting the ground as well as propagation through the earth of upstream coupled energy. The damage mechanism of dominant concern is the missile coming up against and forcibly hitting the shelter wall from the inside, as the shelter moves with the ground. To design for this in a simple MPS [multiple protective shelter] shelter, the missile is given enough space inside the shelter to move before coming up against the shelter wall. This space between missile and shelter is called rattle space, and for shelters several thousand feet distant from a 1-Mt nuclear detonation, typical rattle space is tens of inches. Since at ranges of interest ground shock motions are typically larger in the vertical than horizontal direction, vertical shelters require less concrete than do horizontal shelters, since the inside diameter of the shelter does not need to be as large. In addition, the missile is constructed to be more resilient to motions along its length than transverse to it.
>
> For radiation and thermal effects, since the flux direction on the surface is along the ground, more stringent requirements for the horizontal shelter door are necessary than for the surface-flush vertical door. Electromagnetic pulse effects do not appear to discriminate strongly between horizontal and vertical shelters, although the greater radiation attenuation afforded by the vertical shelter would ease hardness requirements for radiation-induced electromagnetic pulse.

In addition to agreeing with the two other cited sources that airblast effects dominate over nuclear, electromagnetic, and thermal radiation for a vertical silo, the above quotes clarify that airblast-induced ground motion rather than the airblast's overpressure alone is "the damage mechanism of dominant concern." This effect then defines a silo's radius of lethality, making it lethal farther from the point of detonation than any other effect. This is supported with the disclosure that current silos housing U.S. ICBMs cannot be hardened past 2,000 psi of airblast peak overpressure as they currently exist,[45] a level that has not increased as of 2014 according to the RAND Corporation.[46] Hardening a reinforced concrete cylinder or an armored silo door at this level against being crushed cannot be considered a significant challenge,[47] confirming that the structural integrity of a silo would remain intact while the missile inside is destroyed by induced ground motion.

As the *MX Missile Basing* passage also suggests, silo lethality is based on a 1-Mt explosive yield. This is the commonly accepted standard for yield-adjusted hardness that allows the proxy model to accurately calculate silo counterforce probabilities.[48] Thus for warheads with yields below 1 Mt, the proxy model will suggest a more effective counterforce capability than what actually exists.[49] More significant is that *2,000 psi of peak overpressure from a 1-Mt explosion represents the basis from which the lethal nuclear-induced ground motions against U.S. silos must be extracted and is therefore a threshold against which the capabilities of conventional weapons must be compared.*

The stated dominance of the airblast-induced ground motions deserves more consideration, however, as the question remains what ground motions can be included among them. Ground motions induced by nuclear explosions include a complex mixture from several sources, and it is vital to identify those representing the threshold of a silo's survivability. Because while the term airblast-induced clearly identifies which motions are involved, this was not confirmed for some time.

## Airblast-, direct-, and crater-induced ground motion

The ground motions induced by a nuclear explosion are numerous, arrive at the target at different times, and attenuate below lethal levels over varied distances. And even though airblast-induced motions have been identified as the dominant silo threat, it is important to confirm that these motions solely determine the lethal threshold needed for comparison with conventional explosions.

The first distinction to be made between ground motions are those that are *direct* and those that are *indirect*. Direct ground motion is induced by the contents of an exploding bomb without any intermediate effect leading to the ground's motion. In other words, the bomb's exploding contents are directly responsible for moving the ground. Indirectly induced ground motion transpires from the bomb's contents generating an intermediate effect in the air that is then used to induce the ground's motion. Ground motions and stresses caused by weapons detonating at or near the Earth's surface are often defined as one of three varieties of ground shock:[50]

**Airblast-induced shock:** Ground stresses and motions due to the pressure applied by an airblast traveling along the ground as it propagates. This is an indirect effect because the resulting ground motion is an indirect consequence of the bomb's released energy, with the airblast being the intermediary. These motions are typically the high-frequency components of the ground's motions.

**Direct-induced shock:** The ground stresses and motions caused by the initial energy released by the bomb products at the point of burst. Such motions can only be present in near-surface and underground detonations because bomb energies released into the air would result in the growth of shocked air, leading to the indirectly induced ground motion just discussed. Direct shocks occur at a later time than those produced by faster-traveling airblasts.

**Crater-induced shock:** Direct-induced shocks produced by the formation of craters. Because crater formation would follow after the initial direct shocks due to the magnitude of earth excavation involved, these stresses occur at a later time than the others.

These definitions appear in the introduction to the volume *Nuclear Geoplosics Sourcebook, Volume IV, Part I, Empirical Analysis of Ground Motion from Above and Underground Explosions*. It was published in 1979 and is probably still today the most comprehensive survey of ground motion phenomena induced by nuclear explosions. And conceptually, these discussions partly apply to conventional explosions, with those considered in this analysis involving underground direct-induced shock.

There is a physical distinction to be made between ground shock and ground motion. A ground shock results from energy being imparted into the ground by an explosion and travels at a speed known as the seismic velocity.[51] It is simply a stress or pressure wave, i.e., a force, propagating through the ground with a frequency that depends on how energy was transferred into it. It should not be confused with ground motion that is caused by the shock but begins after its passage and has its own distinct motion. Often the velocity associated with ground motion is referred to as particle velocity, which suggests the elemental nature of the ground's local motion. Figure 1 displays the ground shockwave fronts corresponding to those induced by several peak overpressures. The underground shockwaves represent uniform seismic velocities, but higher ground-shock velocities at higher stress levels and with depth are not displayed. At around 300 psi, the ground shockwave at 3,000 ft/s increases in steepness due to earlier higher-velocity shockwaves being superimposed at this location as the air shock slows down. This can often result in higher stresses at these wavefronts than in the air overpressure. And as the air shock slows down further at 100 psi, the ground shockwave at 3,000 ft/s can propagate ahead of the air shock, where it can then be expected to arrive first. This phase is known as "outrunning." This description of the physical phenomena at work in this figure can help clarify a bit of the geometric picture involved, where ground motions will follow after the passage of the ground shocks displayed here.[52]

Figure 2 displays the evolution of nuclear effects shortly after a burst at the surface. Nuclear, electromagnetic, and thermal radiation are accompanied with airblast (overpressure) and airblast-induced ground shock. This occurs roughly 20 ms after detonation. The figure shows the shape of the airblast wave to be a decaying exponential, where the peak is reached rapidly upon arrival and decays with time after its passage. The

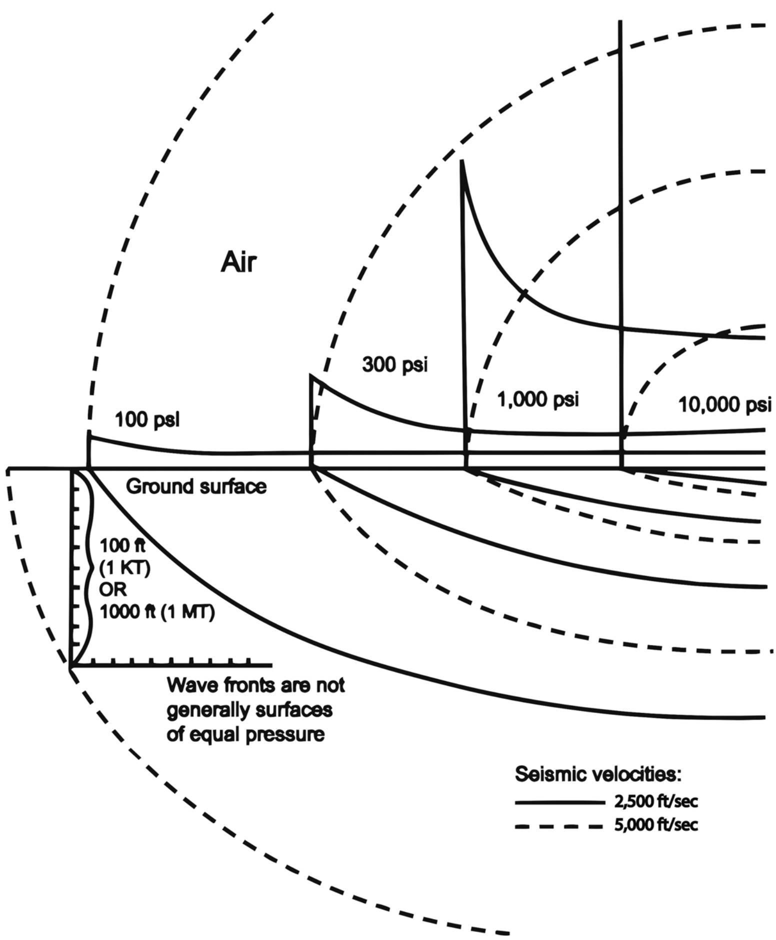


**Figure 1.** Airblast-induced ground shockwave fronts for peak overpressures of 10,000, 1,000, 300, and 100 psi. Modified with permission from *Annual Review of Nuclear and Particle Science*, Volume 18, © 1968 by Annual Reviews, http://www.annualreviews.org.

ground also begins absorbing energy from the bomb's contents resulting in direct-induced ground motion. At this time all ground motions are just a mixture of those induced directly or by airblast. After 3 to 5 seconds, this directly induced motion will form a crater (Figure 3), where massive amounts of debris will be ejected, and a lip will form at the crater's edge.[53]

The question for identifying lethal ground motions is whether any that are direct- or crater-induced contribute. The airblast will arrive first and produce strong vertical and horizontal motions.[54] More direct- and crater-induced motion will follow,[55] the horizontal motions of which were thought might still be—even shortly after the 1981 *MX Missile Basing* study was published—the deciding effect in missile vulnerability.[56] This possibility still existed even after the crater dimensions and direct-induced stress contours in hard rock would appear to almost completely disappear 300 m away from a detonation (Figure 4)[57] and the lethal radius being well outside of that (369 m) for a 1-Mt warhead attacking a 2,000-psi silo (calculated with Equation 3). This would seem to indicate that a silo could be destroyed solely by airblast-induced ground motion.

Yet a clear judgment was not yet possible. First, 2,000 psi was not the settled hardness threshold for a U.S. silo—which would eventually confirm a much larger lethal radius for an attacking 1-Mt warhead than that expected to follow from the original goal of building silos that far exceeded this level. And second, there had been a discrepancy between the theoretical predictions of crater formation (Figure 4)[58] and those obtained empirically from high-explosive data and nuclear tests in the Pacific, with theory consistently predicting smaller craters and less ground motion. Either one of these factors might push the lethal radius into a region

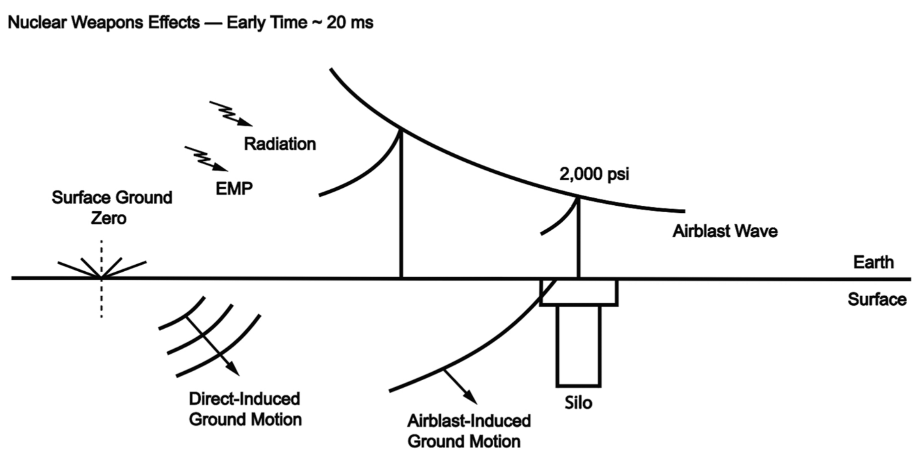


**Figure 2.** The early time nuclear effects that a silo must be hardened against to survive a nuclear explosion. The overpressure threshold for a U.S. silo has been disclosed as 2,000 psi, as displayed here. Modified from U.S. Congress, House Committee on Armed Services, Hearings on Military Posture, 97th Cong., 2nd sess., 1981, 81.

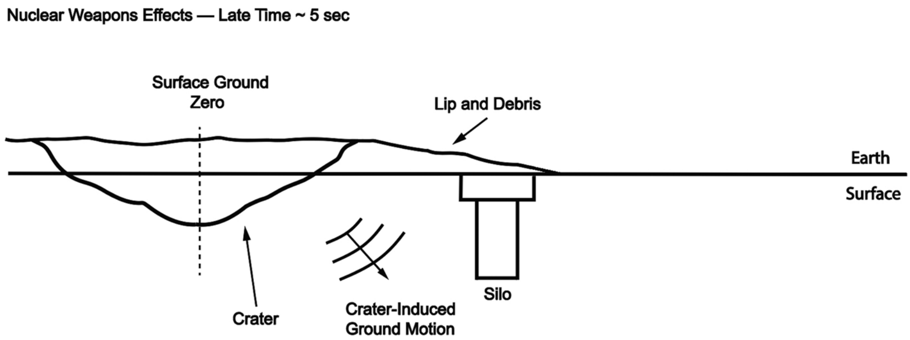


**Figure 3.** The later time nuclear effects that a silo must be hardened against to survive a nuclear explosion. Modified from U.S. Congress, House Committee on Armed Services, Hearings on Military Posture, 97th Cong., 2nd sess., 1981, 81.

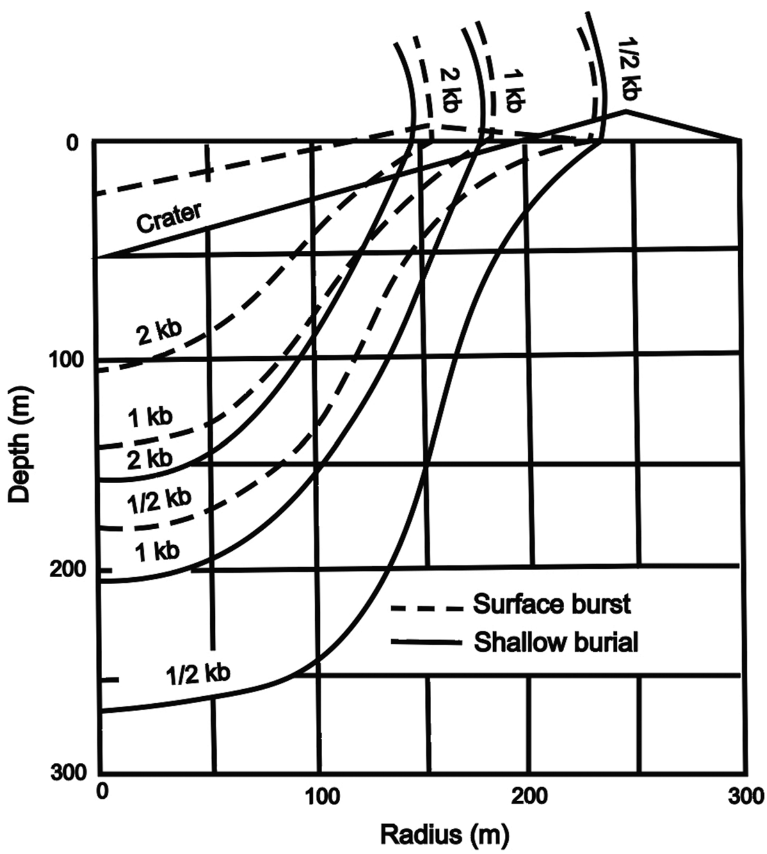


**Figure 4.** The crater dimensions and directly induced shock stress contours in hard rock due to a 1-Mt surface burst and shallow buried burst in kilobars (kb). Modified with permission from *Annual Review of Nuclear and Particle Science*, Volume 18, © 1968 by Annual Reviews, http://www.annualreviews.org.

where airblast- and direct-induced motions are mixed. Later, an underground nuclear "atmospheric" test demonstrated a new method of crater experimentation, and when combined with an exploration of nuclear craters at the Pacific Test Site brought the empirical results into agreement with the existing theory of crater formation.[59]

Today the calculations in Figure 4 are widely accepted, showing that airblast-induced ground motions are capable of destroying the missile inside a silo without any assistance from motion that is directly induced.[60] This silo destruction mechanism then alone has the largest lethal radius of any other, supporting the *MX Missile Basing* study, the judgment reached earlier in this analysis relative to the 2,000-psi limit on silo resistance to airblast overpressure, and other sources cited. Lastly it proves that using cratering dimensions as a proxy for the lethal radius of an attacking warhead is not valid, which is especially tempting when the alternative is using mismatched yields and silo hardnesses as inputs into the overpressure proxy model, typically meaning any yield different than 1 Mt.

The next step is understanding how airblast-induced motions threaten siloed missiles, as this would begin laying out how to model nuclear and conventional lethality, and ultimately, how best to compare them.

## Rattlespace and shock isolation

With missiles being the most vulnerable element within a silo's shock environment, improving ways to protect them has been the subject of multiple research efforts over the years. It turns out that the 2,000-psi hardness limit is determined by two factors not widely discussed in the public domain: shock isolation and rattlespace.

Rattlespace is the gap between the missile's body and the inner walls of the silo, or more likely some element used to attenuate shock that comes into contact with the missile. One supposed way to harden a silo is to increase this gap. The idea is that the silo or missile would then have to be moved farther and/or faster by a nuclear explosion in order for the missile to collide against something with the same force to damage it. The relevant motion may be more complex than this, however, where a missile may increase its speed over more time and rattlespace. These details are unimportant. All silos have roughly the same dimensions, and what matters is there is some rattlespace within which a missile moves that will determine where it is ultimately damaged.

Another method is to improve the silo's shock isolation system, providing a missile with more isolation from the silo's movements. This system could consist of some sort of foam, springs, hydraulic dampers, or elastic cables in the interior of the silo, allowing these elements to absorb a portion of the silo's motion instead of the missile.[61] Figure 5 displays a simple conceptual picture of the rattlespace and shock absorbers used to attenuate the shock transferred to a missile. The shock isolation system usually consists of some elements designed to attenuate the vertical and horizontal shock delivered to the missile. Actual systems inside a silo can obviously be far more sophisticated than what Figure 5 depicts. What matters is being aware of a simple, conceptual understanding of shock isolation and rattlespace that will become important for how conventional and nuclear lethality are compared. The engineering details cannot be known sufficiently to model these systems independently and, in any case, will likely differ among U.S., Russian, and Chinese silos.

Improved hardness could come down to a choice between the cost of a larger silo providing more rattlespace and the cost and complexity of a more effective shock isolation system to better limit the accelerations transmitted to the missile.[62] Sometimes one might be sacrificed at the expense of the other, as a better shock isolation system would take away some of the available rattlespace. This is what happened with the MX missile, the ICBM promoted along with superhard silos to redress U.S. silo vulnerability debated in the 1970s and 80s.[63] It was placed inside a canister to better protect it from nuclear shocks, but this further limited the available rattlespace. A better shock isolation system was added to compensate for this loss, so that the MX and Minutemen missiles were

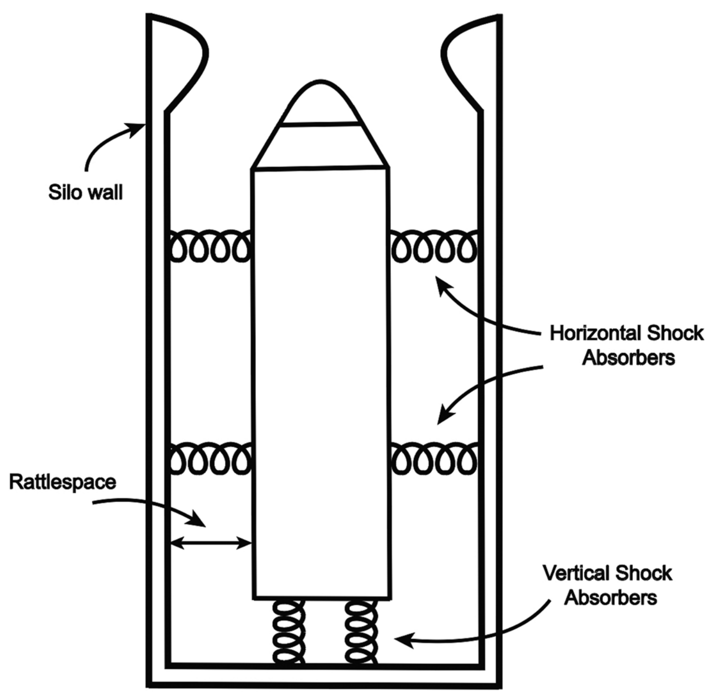


**Figure 5.** A schematic diagram of the rattlespace and horizontal and vertical shock absorbers between a missile and the silo's inner walls and floor.

each protected by the same silo hardness in the end. The combination of rattle space and shock isolation available to each missile somehow made their protection the same, or this is what was claimed.[64]

Engineers involved in the MX missile project often made a distinction in 1980s news interviews between the hardness of the silo structure and "hardness" improvements needed in the shock isolation system, offering their view that the shock isolation system was the weaker, and thus more crucial, element.[65] This challenge is reflected in Figure 6, which displays the options for permitting the missile to survive a higher acceleration, i.e., a force, inside a silo caused by the ground motions surrounding it. Here an improved canister (used for the MX missile but not the Minuteman)[66] or a more hardened missile can shrink the rattlespace and permit the allowable acceleration to increase, while an optimized shock isolation system (SIS in the figure) also shrinks the rattlespace but leaves the acceleration unaffected. As a consequence of discovered or accepted limits related to efforts here, hardness is limited to 2,000 psi for U.S. silos.

Some valuable insights into a Chinese perspective on shock isolation are provided in the Appendix. Rocket engineers performed a finite element method analysis to model which shock isolation system would be the most effective at limiting accelerations felt by the siloed missile. They used many of the same ground motion parameters used in this analysis,

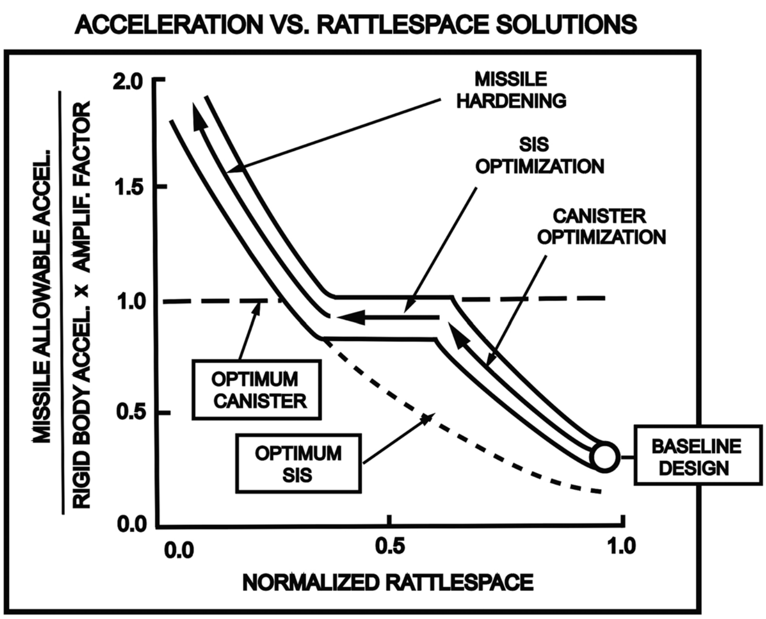


**Figure 6.** The maximum allowable acceleration for a missile inside a silo as a function of normalized rattlespace as a missile canister, shock isolation system, and further missile hardening are added to provide more protection to the missile. Only the now retired MX missile had a protective canister; current Minutemen III missiles do not. Modified from "Shock and Vibration Bulletin," 1987.

confirming a similar perspective with how to translate ground motion into responses by the elements inside a silo. They also found crucially that vertical motion can be neglected in assessing missile survivability.

## Silo hardness in the United States, Russia, and China

With the hardness limit of U.S. silos already established, those for Russian and Chinese silos need to be referenced as well. As discussed, these limits will define the lethal ground motions induced by nuclear explosions that underground conventional explosions must be compared to.

Estimates for China's silos are, of course, highly uncertain. But Chinese research has modeled ballistic missiles with a diameter of 3 m,[67] making them roughly consistent with the diameters of the DF-5 ballistic missile.[68] These dimensions are consistent with the 3 m diameters of Russia's largest ICBMs, the RS-28 Sarmat[69] and the SS-18 "Satan."[70] The silo diameters that house both are about 6 m,[71] which is similar to an early estimate of the diameter of China's new silos said to resemble those in Russia.[72]

Today Russia likely possesses slightly fewer than 150 ICBM silos,[73] down from a 2015 Russian projection that assumed by 2020 they would maintain around 180.[74] Archival research reported that Russian silos in 1985 had a maximum hardness of 1,500 psi, with some missiles still around today possibly housed in silos of 900 psi.[75] Unless China has developed a more sophisticated shock isolation system than Russia, there is little reason to believe that the hardness of their silos exceeds these levels. This is especially true given the upper limit of 2,000 psi on current U.S. Minutemen silos discovered after years spent attempting to harden them, an experience China lacks. The silo hardnesses for all three countries are displayed in Table 1, with an assumption that there may be some older Chinese silos with hardnesses around 450 psi. This is nothing more than speculation, however, if perhaps some older Russian-designed silos remain. The safer assumption is that there is nothing below 1,500 psi, with there being little evidence or reason in the public domain to believe they exceed this level yet. Of course, as this paper has endeavored to make clear, several factors play into how these numbers are determined.

## The need to compare conventional with nuclear lethality

Comparing conventional with nuclear lethality arises from the limited information available on the conventional side to proceed with an independent, stand-alone assessment. Perhaps the most crucial among them is that the magnitude of force needed to lethally damage the siloed missile is not knowable.

**Table 1.** The proxy values of silo hardness in psi (United States, Russia, and China).

| Country | Silo hardness (psi) |
|---|---|
| United States[76] | 2,000 |
| Russia[77] | 900–1,500 |
| China | 450–1,500 |

The advantage in choosing a comparison is that the known silo hardnesses in the United States, Russia, and China provide enough information on what is lethal so that what is lacking for a technically valid assessment is adequately compensated for. Details about how exactly the shock isolation system is designed are not available, and no model of a missile moving inside a silo after an explosion should be considered accurate without these details. Using a comparison allows these limitations to be bypassed.

What is available is the free-field ground motion, that induced by a nuclear airblast at the surface, which determines the silo hardnesses and represents the dominant destruction mechanism among all nuclear effects that accompany a nuclear explosion. These ground motions will move a silo along with them and must be compared with those directly induced by underground conventional explosions that can now accompany earth-penetrating long-range conventional weapons. A careful comparison between the relevant nuclear and conventional ground motions and their impact on the shock isolation system should reveal the maximum distance (radius of lethality) from a silo at which a conventional weapon can destroy the missile inside.

In the sections that follow, peak motion parameters were used to model ground motion. They are used in almost all calculations and models that appear in the technical literature rather than less reliable or inaccessible time-dependent waveforms. The fundamental motions of displacement, velocity, and acceleration are said to be more accurately obtained by using correlations of peak values of these parameters as a function of range from the detonation.[78]

This nuclear–conventional comparison also cannot be applied across the entire range of the radius of lethality for a conventional weapon. If a conventional weapon directly impacts a silo cover composed of armored steel, it will no longer be able to induce ground motions by detonating underground. The question becomes whether it is able to fully penetrate the silo cover or partial penetration accompanied by a detonation is adequate to destroy the missile housed in the silo. There may also be a chance that some direct impact could move the silo sufficiently to damage the missile. Of further relevance is the ratio of the area of lethality determined by induced ground motions to that of the silo cover itself. This would provide some idea of how much more vulnerable the silo is beyond only a direct impact, i.e., its greater kill probability. A more detailed discussion of this prospect is provided in the Appendix.

The comparison that follows could also benefit from confirmation by a finite element method analysis. This is discussed in a later section, but there may be limits to what such an effort could confirm given the nature of why this nuclear–conventional comparison was necessary.

## Modeling nuclear lethality

The equations used to calculate the peak displacement, velocity, and acceleration along the length of a silo from airblast-induced nuclear surface explosions are displayed in this section. They were developed with data obtained from an atmospheric nuclear test at the Frenchman Flat lake bed at the Nevada Test Site (NTS) in 1957. A more lengthy discussion supporting why they are valid for the purposes of this assessment is provided in the Appendix.

The peak displacement $d$ (in inches here, not feet) can be calculated as a function of the peak overpressure $P_o$, i.e., the hardness of a missile silo, explosive yield $Y$, and depth $z$ below the surface:

$$d = 7\left(\frac{P_o}{100\,\text{psi}}\right)^{0.78}\left(\frac{Y}{1\,\text{Mt}}\right)^{\alpha} e^{-0.0085(z)}\ \text{(in.)}\ \text{for}\ 0 \le z \le 100\,\text{ft} \tag{5}$$

where

$$\alpha = 0.15\left(\frac{P_o}{100\,\text{psi}}\right)^{0.16} \tag{6}$$

The particle velocity $v$ (in inches per second) can be calculated with

$$v = 50\left(\frac{P_o}{100\,\text{psi}}\right)^{0.95}\left(\frac{Y}{1\,\text{Mt}}\right)^{\beta} e^{-0.0085(z-30)}\ \text{(in./sec.)}\ \text{for}\ 30 \le z \le 100\,\text{ft} \tag{7}$$

where

$$\beta = 0.07\left(\frac{P_o}{100\,\text{psi}}\right)^{0.36} \tag{8}$$

and the acceleration $a_v$ (in units of gravity, $g$) with

$$a_v = 1.5 \cdot P_o \cdot z^{-0.83}\ (g) \tag{9}$$

These models can be used to calculate the peak ground motions at any depth below the surface along the side of a silo. Those lethal to it will be obtained by inserting the appropriate silo hardness in for $P_o$. The Appendix explains why they are appropriate to use for silo hardnesses as high as 2,000 psi when they were only developed for those up to 1,000 psi. It also needs to be stressed that as $P_o$ increases, these motions will attenuate more with depth than these models suggest. The models are then are an overestimate of the ground motions at the level of thousands of psi of interest here, presenting a higher standard that conventional weapons must reach to be considered lethal to a silo.

## Modeling conventional lethality

By comparison with nuclear-induced ground motions, those resulting from conventional explosions are more predictable. This is no doubt aided by the greater number of underground high-explosive tests conducted compared to atmospheric nuclear tests, very few of which measured ground motion.

The most widely accepted paper for its models of ground shock from earth-penetrating conventional weapons is a short update to a well-known U.S. Army design manual, TM 5-855-1. It was written by James Drake and Charles Little, and they analyzed more than 50 tests conducted over many years to present empirical prediction equations for ground motion.[79] General waveform parameters such as displacement, impulse, and acceleration may be derived from the time-dependent expressions for the stress $P(t)$, i.e., pressure, and particle velocity $V(t)$ provided in the paper, but as mentioned earlier these waveforms are typically too complex and inaccurate for any useful ground motion to be estimated in a specific case. It is better instead to rely on the peak values of these measured parameters.[80]

Drake and Little provided the empirically determined formulas[81]:

$$P_o = f \cdot (\rho c) \cdot 160 \cdot \left( \frac{R}{Y^{1/3}} \right)^{-n} \tag{10}$$

$$V_o = f \cdot 160 \cdot \left( \frac{R}{Y^{1/3}} \right)^{-n} \tag{11}$$

$$a_o \cdot Y^{1/3} = f \cdot 50 \cdot c \cdot \left( \frac{R}{Y^{1/3}} \right)^{-n-1} \tag{12}$$

$$\frac{d_o}{Y^{1/3}} = f \cdot 500 \cdot \frac{1}{c}\left(\frac{R}{Y^{1/3}}\right)^{-n+1} \tag{13}$$

where $P_o$ is the peak stress, $V_o$ the peak particle velocity, $a_o$ the peak acceleration in g's, $d_o$ the peak displacement in feet, $R$ the distance from the explosion in feet, $f$ a coupling factor for near surface detonations, and $n$ an attenuation coefficient equal to about 2.5 in typical soil.[82]

The seismic velocity $c$ in ft/s can be obtained from any number of easily accessible tables, but its value does not remain constant with depth. However, given that the nuclear-induced ground motions in the previous section were for test data from the Frenchman Flat at the NTS, using the seismic velocity from that location is the most appropriate for the most accurate comparison. An average value was calculated for that location that took into account three different layers, each with different seismic velocities, and found that it equaled 2,367.91 ft/s.[83] It will become clear that for the best comparison between nuclear and conventional ground motion, the seismic velocity will no longer be relevant in the conventional expression, but it is worth keeping in mind this accurate comparison.

Drake and Little define the coupling factor $f$ as "the ratio of the ground shock magnitude from [a] partially to shallow buried weapon to the ground shock magnitude from a fully buried burst in the same medium."[84] Data was extracted from a graph that plots $f$ as a function of the scaled depth of burst ($d / Y^{1/3}$) and is presented in Table 2.[85] Here $f$ is shown to reach its maximum of 1 when the scaled depth of burst reaches 1.4. For 1,000 lb of TNT-equivalent explosive material, this is 14 ft. Such a bomb must reach this depth for all of its resulting ground shock to maximally couple to the ground.

An alternative explanation for $f$, however, explains that instead of penetrating to a maximum depth necessary for maximum coupling, there exists an optimal detonation depth that is less than optimal at depths above or below it. If the depth is too shallow, the bomb's explosive gases will break up the earth and eject it into the air forming a crater. But if the pressure is just below sufficient to lift the soil, a camouflet will form, which is considered optimal. This occurs when the scaled depth of burst equals $(2\,\text{ft/lb})^{1/3}$ in silty clay or more specifically:

$$\text{Optimal depth of burst} = 2 \cdot Y^{1/3}\ (\text{ft.}) \tag{14}$$

can be used to calculate the optimal depth of detonation.[86] One thousand pounds of TNT would need to detonate at 20 ft in such a medium to ensure the formation of a camouflet. At depths of burial greater than

**Table 2.** How the coupling factor $f$ varies with the scaled depth of burst in soil.

| Scaled depth of burst ($d/Y^{1/3}$) in ft/lb$^{1/3}$ | 0 | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 |
|---|---|---|---|---|---|---|---|---|
| Ground shock coupling factor, $f$ | 0.4 | 0.6 | 0.7 | 0.8 | 0.88 | 0.92 | 0.97 | 1.0 |

*Note.* From Drake and Little, *Ground Shock from Penetrating Conventional Weapons*.

optimum, there would be a slight reduction in $P_o$ because the reflecting shock from the underside of the soil begins to diminish.[87]

It is not obvious to the author whether the depth to optimize $f$ is significantly better than merely reaching a depth to maximally couple the burst's ground shock. In any case, the optimal depth still appears to be within reach for both the Tomahawk and the JASSM, with deeper penetration possible if any of the following are true: (1) the density of the warhead and its explosive material exceeds hard steel; (2) the length of the warhead exceeds 6 ft (the likely length of the JASSM's J-1000 warhead); (3) a terminal velocity greater than 1,000 m/s is possible without significantly eroding the warhead; or (4) penetration into geology softer than reinforced concrete is required.[88,89] All of these appear very likely, with deeper penetration of interest due to the possibility of a conventional weapon detonating closer to the key location along a silo's length where a lethal force is transmitted to the siloed missile.

Of further relevance is that the energy of explosive material in modern warheads greatly exceeds that of TNT. Some specifics are available publicly, with that for the JASSM disclosed as yielding a TNT equivalency, the ratio of a material's explosive energy to that of TNT, of 1.65. It is likely a safe assumption that all U.S. missiles have now been supplied with a similar material, but to avoid detailed focus here the TNT equivalence was set at 1.5 for both the Tomahawk and the JASSM.[90]

With both the nuclear and conventional models of explosion-induced ground motion presented here, the next step is understanding how to compare these motions to assess silo vulnerability to conventional weapons.

## Comparing nuclear with conventional

Relating nuclear and conventional ground motions involves several factors. The two possess different geometric profiles of the key parameters that vary with depth. How these motions are compared and translated into responses by the shock isolation system that determine what acceleration a missile experiences deserves careful consideration. It is also important to confirm that the relevant silo response modes between the motions are sufficiently understood to allow for an accurate comparison. The models presented in the previous sections also calculate only peak motions (displacement, velocity, and acceleration), so there may be some time dependence of the motions to take into account. This section describes how this comparison was made and which parameters this analysis is sensitive to.

## Horizontal versus vertical motion

Because the dominant ground motions are induced by the airblast moving along the surface, the resulting motions will be greatest in the vertical direction but will contain significant horizontal components. On the conventional side, vertical motions will exhibit a symmetry above and below the detonation point, allowing them to mostly cancel and be neglected. They will also play no role in the lethal acceleration imparted to the missile in the shock isolation system's response to either a nuclear or conventional explosion. The question is how to determine the horizontal components of the nuclear motion.

One way determined empirically from data at the Frenchman Flat site is that the ratio of horizontal to vertical displacement is equal to the tangent of the angle between the peak stress shock and the Earth's surface. It usually varies from 0.1 to 0.5 and increases as the angle does.[91] The ratio of horizontal to vertical acceleration is obtained by taking the tangent of the angle between the compressional wave and the Earth's surface. The horizontal and vertical acceleration are considered equal if this quantity is greater than one.[92] The compressional wave will travel faster than the peak stress front, and this leads to the two ground shocks traveling at slightly different angles with respect to the Earth's surface. These different angles explain why horizontal acceleration is typically a larger fraction of vertical acceleration than horizontal displacement is of vertical displacement.[93] There are obviously uncertainties here and these horizontal-to-vertical ratios will vary with depth, but these angles of inclination are considered a valid geometric approach roughly consistent with empirical data for relating horizontal to vertical motion.[94] It should be understood that the complexity of different angles, velocities, and varieties of pressure pulses prevents a simple relationship from being obtained. For this reason, a simple horizontal to vertical relation that also happens to be the most commonly used was adopted.

The section "Modeling nuclear lethality" in the Appendix provides more analysis and the sources relied on for the judgements stated here, but the most commonly agreed-on relationships state that horizontal displacement due to nuclear airblast-induced motion is one-half that of vertical. And that the horizontal peak velocity and acceleration are equal to those in the vertical direction.

For nuclear airblast-induced ground motion:

$$\text{Horizontal Peak Displacement} = \frac{1}{2}\text{Vertical Peak Displacement}$$

$$\text{Horizontal Peak Velocity \& Acceleration} = \text{Vertical Peak Velocity \& Acceleration}$$

The one-half relationship for displacement is conservative when compared to the tangent procedure just mentioned, thus placing a greater burden on conventional capabilities, in the sense that some suggest the horizontal motions may not be this high relative to those that are vertical.[95] These relationships are also not expected in most instances to remain constant with depth. While nothing higher than one-half has been reported for the horizontal displacement, modeling the sensitivity between one-third and two-thirds of vertical seems appropriate. And considering a horizontal velocity and acceleration down to two-thirds of these vertical parameters appears reasonable as well. This will test how sensitive the final results are to these judgements.

## Shock isolation model

The next task is to compare how ground motions along the silo's length affect the response of the shock isolation system and the acceleration experienced by the missile. A fuller analysis and derivation of the results presented here appears in the section "A simple model of shock isolation" in the Appendix. A discussion of the silo response modes relevant for providing further support to these results follows in the next section.

The need to consider the response of a shock isolation system explains why a nuclear–conventional comparison is necessary. With the engineering details of the exact system in the silo's interior unknown, no alternative exists other than a comparison that can account for unknown elements that are vital to this assessment. The idea is that if some combination of ground motions induces the shock isolation system to respond in the same way, the missile will experience the same lethal acceleration from both nuclear and conventional explosions.

Determining how to approach this becomes easier with clarity about the shock isolation response to ground shocks in the vertical versus horizontal direction. Valuable information here was provided during 1984 congressional testimony in which it was revealed that the limiting factor in silo hardness was the shock isolation for attenuating the horizontal motion of the missile inside the silo. The top and bottom of the silo will not come into play as surfaces that will rattle against the missile, except at airblast peak overpressure levels beyond the 2,000-psi threshold.[96] Thus a missile will be destroyed by its horizontal motion inside a silo before its survivability is put at risk by any in the vertical direction. The same claim was made in a Chinese analysis of shock isolation discussed in the Appendix.

This makes intuitive sense even though it is well known that ground motions induced by a nuclear airblast are larger in the vertical than horizontal direction. First, the MX missile was 72 ft in length[97] compared to that of just under 60 for the Minuteman III.[98] If vertical rattle space or shock isolation was a limiting factor, a longer missile would not have been proposed. And if more vertical space was needed to accommodate a longer missile, it seems likely that lengthening a silo by extending its structure farther into the ground would be a comparatively easier task than expanding a silo's diameter. This later effort would require the construction of an entirely different silo, and even then the increased rattle space may come with other complications for guaranteeing survivability due to the surrounding support equipment to keep the missile upright when experiencing ground shocks or otherwise properly aligned in order to launch cleanly and accurately to the intended target. And lastly, a longer silo would likely experience less vertical ground motion because the ground it rests on will move less at a greater depth.

A simple spring system was modeled that considered only the horizontal ground motions that impinge on the silo's outer wall and compress the spring accordingly. The spring in this case could be whatever element in the shock isolation system absorbs the silo's motion to best protect the missile; a spring is a proxy for whatever that may be. By setting the spring's compression equal between nuclear and conventional motions for the time shortly after the silo responds to the ground motion, you equate how the shock isolation system responds between the two and assume that the lethal force necessary to destroy the missile is the same between them. The mathematical derivation following from this basis found that the spring's compressions were equal when $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ were equal.

$$\underset{\text{Nuclear}}{\left[\text{peak accel.} \times \text{peak displ.}\right]^{1/2}} = \underset{\text{Conventional}}{\left[\text{peak accel.} \times \text{peak displ.}\right]^{1/2}}$$

This relationship is true only during the early moments of a silo's response because nuclear and conventional ground motions have different peak displacements, velocities, and accelerations. The compression becomes too difficult to equate between the two as time progresses. The idea is you determine some key parameters when it is reasonable to do so without knowing exactly what state the shock isolation is in or where exactly the missile is when it is destroyed. In general, the compression's magnitude will increase more on the nuclear side (due to a larger peak displacement) and its velocity more on the conventional side. But the shock isolations system's early compression is roughly equivalent between the two motions when $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ is equal, which is also the peak velocity. The peak velocity represented in Equation 11 is obtained from this expression in the conventional case. But using two independent parameters here provides a stronger result given the uncertainties on the nuclear side, especially with peak velocity (see "Modeling nuclear lethality" in the Appendix).

The quantity $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ is plotted as a function of depth for both nuclear and conventional motion in Figure 7. The obvious difference in profiles raises the question of how the shock isolation system might respond differently in each case. The safest or most conservative option for considering conventional lethality is to take the average of $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ and set the detonation point for a conventional weapon a distance away from the silo where nuclear and conventional averages are equal. Figure 7 displays profiles whose averages are equal, a distance 19.5 ft away from a 1,500-psi silo in the case of a Tomahawk cruise missile it turns out. This method appears a bit crude and should be confirmed with a finite element analysis, but the next section provides more robust justification for this approach.

The model constructed uses the expressions for nuclear motion (Equations 5, 6, and 9) and those for conventional motion (Equations 12 and 13) to construct expressions for $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$. Some basic geometry and trigonometry are used on the conventional side to obtain the horizontal motion, and the peak nuclear displacement must be divided by 12 to convert from inches to feet. Integrating both sides from 5 ft (where the relevant motion is likely to start on the nuclear side) to 90 ft (the known length of a U.S. ICBM Minuteman III silo) provides

$$\overbrace{\int_5^{90} 0.109 \cdot P_o^{0.89} \cdot Y_{nuc}^{\gamma} \cdot z^{-0.415} e^{-0.00425(z)} dz}^{\text{Nuclear}} = \overbrace{\int_5^{90} 158.1 \cdot Y_{conv}^{\frac{5}{6}} \cdot x\left(x + (z-h)^2\right)^{-\frac{7}{4}} dz}^{\text{Conventional}} \quad (15)$$

where $P_o$ is the peak overpressure of the airblast (i.e., silo hardness in psi), $\gamma = 0.0359 \cdot P_o^{0.16}$, $z$ is the local depth of the ground motion, $Y_{nuc}$ is the nuclear explosive yield in Mt, $Y_{conv}$ is the conventional yield in pounds, $x$ is the horizontal range from the silo's outer wall to point of detonation underground, and $h$ is the depth of the detonation (calculated from Equation 14). The nuclear case is straightforward to determine, but the conventional case on depends on $x$. A range of $x$ values can be used to calculate several integrals and then a fit to this data determines where a conventional detonation of some yield intersects with nuclear for a particular silo hardness.[99] The results of these calculations are displayed in Figure 9 in the section "Determining conventional lethality."

This comparison between nuclear and conventional motions has some sensitivities to a few parameters, but it is a simple, clear basis from which sensitivities can be considered.

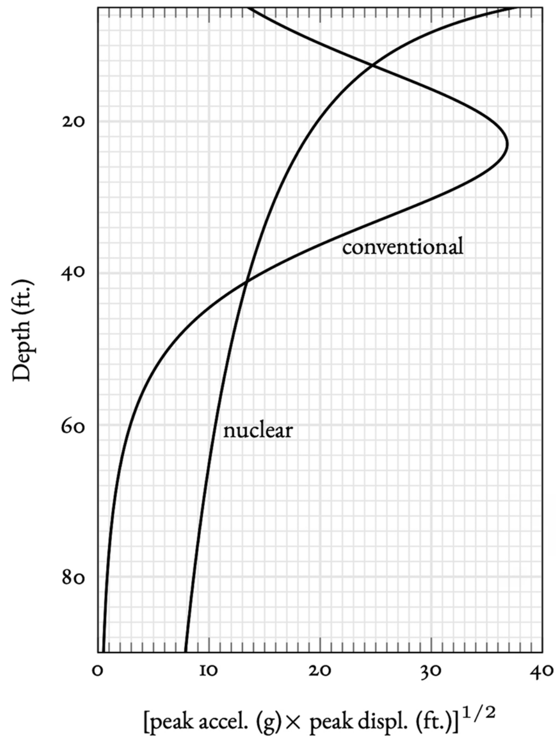


**Figure 7.** The quantity [peak accel. × peak displ.]$^{1/2}$ plotted as it varies with depth for a conventional weapon of 1,500-lb yield (equivalent to a U.S. Tomahawk cruise missile) detonated 23 ft underground and 19.5 ft away from a 1,500-psi silo, along with the same quantity for a nuclear weapon needed to destroy the siloed missile. The value plotted represents the ground motion just outside the wall of the silo.

## Silo response modes

One question that follows from the different profiles in Figure 7 is whether the peak motions underpinning these profiles are an accurate reflection of a silo's response. In addition to this question, this section also accounts for the time dependence of ground motion that was disregarded with the choice to use peak motions. Two different finite element analyses were used to support the analysis that follows here, but another such analysis could confirm this discussion.

A 1965 U.S. government report on design procedures for shock isolation systems due to directly transmitted ground shock modified a well-known finite element analysis to model the response modes of an elastic cylinder in an acoustic medium.[100] It used a modal analysis that essentially broke down the cylinder's response into two motions: the rigid-body motion of the shell and the elastic shell's response. The modification of the model used as a basis here permitted more sophisticated incident waveforms and soil models.[101]

The different response modes for a cylinder's cross-section under a pressure pulse are displayed in Figure 8. All six figures display the displacement (labeled $Q_n$, where n indicates the response mode) as a function of time. Figure 8a displays a response that is identical to the total response in Figure 8f. The report where this figure appeared noted that with regard to the rigid-body response: "The shell rapidly accelerates to the particle velocity associated with the stress wave in the free field. Total displacement is substantially the same as the free field."[102] This indicates that the difference in nuclear versus conventional ground motion profiles displayed in Figure 7 will not translate into some other dilation or bending mode response among those displayed in Figure 8. The silo's response will approximate that of the calculated free-field particle velocity and displacement for both, just like a rigid body would predict. The maximum slope in Figure 8a would be the peak particle velocity according to this description.[103]

Yet this raises the first key question: will the silo reach this terminal peak velocity (i.e., $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$) in response to a conventional blast with a shorter pulse duration? The report mentioned that a step pulse must be approximately two silo diameters long to reach peak particle velocity in the free field. For an exponentially decaying pulse of relevance here, this may depend on the magnitude of the stress pulse permitted at the end of two silo diameters, but a later finite element model considered how all cylinder response modes would affect the time needed to reach the maximum velocity.

This new model also used an elastic cylinder in an acoustic medium, but included the stretching effects (i.e., modes other than the rigid-body mode) felt by the cylinder to conclude that the cylinder's maximum velocity would be reached in less than one-half the time needed for the pulse to travel the length of the cylinder's radius.[104] Such a duration is less than one-eighth of two silo diameters, with most maximum responses for exponentially decaying pulses expected to occur before the pulse transited across the cylinder's radius.[105] While this analysis is for an acoustic medium and not a more sophisticated soil variety, this does suggest that silos will reach the peak particle ground velocities produced by conventional explosions. Another finite element analysis should confirm this.

Yet such an analysis cannot confirm that comparing averages of $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ along a silo's length is too conservative for assessing conventional lethality. And so this raises the second key question: is Equation 15 too conservative a comparison? This may be true if more knowledge was available about which element inside a silo is compressed to attenuate the shock transmitted to the missile. It is likely that due to the response modes of an elastic cylinder considered in this section, a

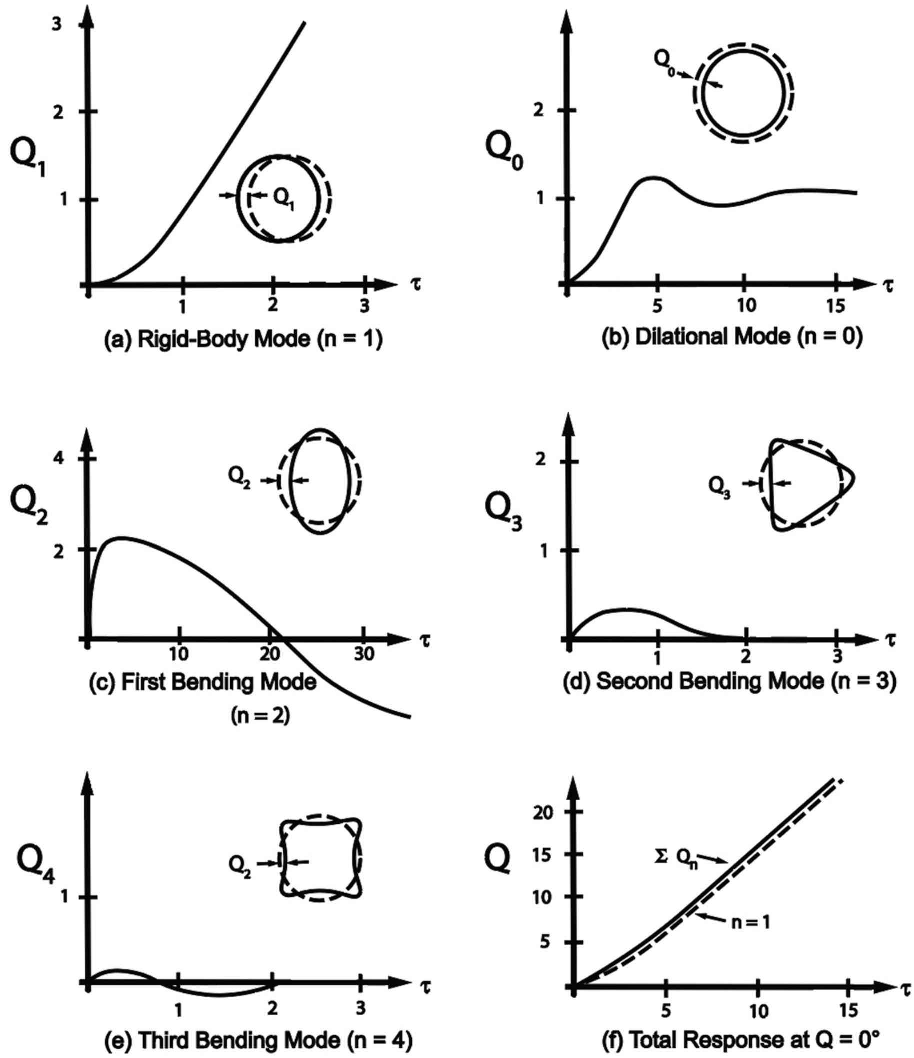


**Figure 8.** The displacement ($Q_n$, n = 0,1,2,3,…) response modes of a cylinder's cross-section to a pressure step pulse plotted as function of time. The main features here are similar to a decaying exponential pulse that best represents the time-varying ground shock of a nuclear or conventional explosion. Modified from F. Finlayson et al., "Design Procedures for Shock Isolation Systems of Underground Protective Structures Volume II: Structure Interior Motions Due to Directly Transmitted Ground Shock."

penetrating conventional cruise missile can detonate closer to the key element of the shock isolation system and cause it to respond in a manner more lethal to a missile than the detonation point obtained by the calculated average performed here. Other advantages for conventional weapons may appear if they are able to penetrate deeper to where they need to detonate near this element. The average comparison with nuclear in Equation 15 would improve favorably somewhat assuming that the explosive coupling did not depart significantly from optimal, but the detonation point might move even farther away from the silo as a consequence of

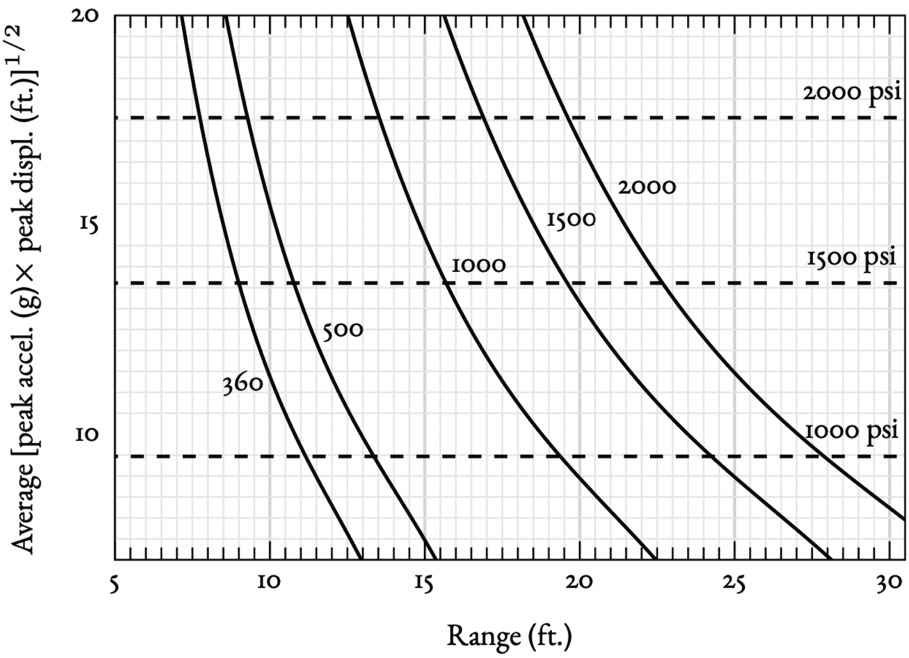


**Figure 9.** The average of the quantity [peak accel. × peak displ.]$^{1/2}$ for conventional explosive yields of 360, 500, 1,000, 1,500, and 2,000 lb plotted as a function of range from the silo's outer wall. This quantity representing the lethal ground motions induced by nuclear explosions appear in the dashed lines for 1,000-, 1,500-, and 2,000-psi silo hardnesses. The range at which a conventional detonation is able to destroy the missile housed inside a silo is represented by the intersection of the x-axis value with these conventional and nuclear lines.

affecting the shock isolation system similarly to a nuclear explosion at the surface. It is unknown how this might work with the limited knowledge available publicly, but the suggestion that silo responses follow free-field ground motions does make this entire lethality assessment likely a conservative one. This is particularly true assuming that the key shock isolation element that needs to be affected is located in the top half of the silo's interior, more easily accessible to an earth-penetrating conventional weapon. If considering a particular element is not exactly right, then a broader area along a silo that will transmit a force to the missile's center where it will collide with the cage and break apart may be more accurate. It does not seem likely that the ground motions next to the bottom half of the silo will be nearly as important as those in the top due to the suspended cage that houses a U.S. ICBM and the room needed in the bottom half for a shock isolation system under the missile to attenuate vertical ground shock.

This possibility, however, cannot discount the chance that the silo does not reach the peak velocity due to a conventional explosion's shorter pulse duration. While this discussion has made clear that this does not appear likely, the sensitivity of this analysis to it is considered later.

## Ranges of parameter values

While the comparison in Equation 15 represents a solidly supported, simple, and transparent comparison, the parameters to which it is sensitive need to be tested. In addition to horizontal and vertical ground motions, a smaller peak velocity on the conventional side needs to be considered in the silo's response. And while the U.S. government does state that the CEPs on its precision munitions are less than 3 m, calculating conventional lethality from 2 to 4 m seems reasonable. The maximum penetration depth is also likely significantly greater than what is considered optimal, but should this capability be less than optimal or adversely affected by some countermeasure, the sensitivity of the analysis to this parameter should be known. The parameters this analysis may be sensitive to are displayed along with their relevant range of values in Table 3. Equation 15 can be adjusted accordingly.

One last factor worth mentioning is that the time dependence considered in the silo's response in reaching peak velocity is unlikely to differ in any meaningful way between nuclear and conventional motion given the evidence that maximum velocity will be reached over a short time. The caveat here is that conventional ground motions will still move at a higher frequency given their higher peak accelerations and smaller peak displacements relative to nuclear motions. This means that the shock isolation system may be oscillating between the calculated peak motions, albeit with decaying amplitude over time, at a higher rate than nuclear thus enhancing the lethality of conventional motion. There is not much to be done with this considering the uncertainty with how this could be included, yet it is a factor this analysis could be sensitive to. And most importantly, as already mentioned, if a conventional weapon is able to detonate underground near the crucial shock isolation element or at some location where lethal ground motions are more readily induced, the comparison made in Equation 15 may be understating the lethality of conventional weapons.

## Determining conventional lethality

With the establishment of the needed nuclear–conventional comparison, this analysis moves toward how to implement it. What follows is a procedure that could be used to determine the lethality of any underground conventional explosion against a silo of any hardness.

The goal here is straightforward: determine the distance from a silo's outer wall at which a conventional weapon would need to detonate underground so that the nuclear and conventional sides of Equation 15 are equal. While sensitivities to the parameters in Table 3 will be considered, Equation 15 provides the most solidly supported comparison, with the most likely values of the relevant parameters previously discussed embedded therein. Figure 9 plots the average of $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ for conventional detonations as a function of range from the silo with the same averages for nuclear detonations needed to destroy 1,000-, 1,500-, and 2,000-psi silos represented by the dashed lines.[106] The range of the detonation $x$ needed for Equation 15 is found by the x-value of the intersection between conventional and nuclear. This range added on to the radius of the silo equals the lethal radius (LR from Equation 2) reprinted here:

$$\text{SSKP} = 1 - 0.5^{\left(\frac{\text{LR}}{\text{CEP}}\right)^2} \tag{16}$$

The missile's CEP is then used to calculate the SSKP of a conventional weapon against a missile silo of a particular hardness.

This simple procedure was followed for two U.S. long-range precision cruise missiles: the Tomahawk and the JASSM. The most recent Block IV/V version of the Tomahawk contains 1,000 lb of high explosive and has a range of 1,600 km.[107] This yield is scaled up to 1,500 lb and its optimal penetration depth, as calculated from Equation 14, is 23 ft. There are two varieties of the JASSM: the JASSM and JASSM-ER, with respective ranges of 370 and 1,000 km.[108] Both have a warhead filled with 240 lb of AFX-757 high explosive,[109] which scales up to a yield 360 lb. Their optimum penetration depth is 16 ft.

Using Figure 9, the range obtained by the intersection of the Tomahawk with a 1,500-psi silo (the likely maximum hardness of silos in Russia and China) is 19.5 ft, and that for the JASSM is approximately 9 ft. The nuclear and conventional profiles for the quantity $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ along the outer wall of this silo are displayed in Figure 7 for the Tomahawk and Figure 10 for the JASSM. Similar plots can be obtained for any silo hardness or conventional explosive yield. For a Russian or Chinese silo with a 3-m radius (9.84 ft), the lethal radius for a Tomahawk and JASSM become 8.93 and 5.61 m (29.34 and 18.84 ft), respectively. For a U.S. Minuteman III silo, the radius to be added to ranges pulled from Figure

**Table 3.** Parameters that this analysis may be sensitive to and their relevant ranges of values.

| Parameter | Range of values |
|---|---|
| Nuclear-induced horizontal displacement (% of vertical) | 1/3–2/3 |
| Nuclear-induced horizontal velocity (% of vertical) | 2/3–1 |
| Nuclear-induced horizontal acceleration (% of vertical) | 2/3–1 |
| Silo peak velocity (% of peak ground velocity) | 1/2–1 |
| CEP (m) | 2–4 |
| Penetration depth (ft) | 10–optimal |

*Note.* CEP = circular error probable.

9 is only 1.83 m (6 ft), resulting in respective lethal radius of 6.95 and 4.19 m (22.8 and 13.74 ft) for the Tomahawk and JASSM.[110]

With the U.S. government claiming an CEP accuracy less than 3 m for its precision-guided munitions, both 2- and 3-m accuracies were used in calculations here and displayed in Tables 4 and 5.[111] The Tomahawk is capable of destroying a 1,500-psi silo with either 99.8% or 100% probability depending on its CEP, with the JASSM's SSKP equal to 92.1% or 99.7% depending on the same. The probabilities for these U.S. systems against U.S. 2,000-psi silos were 97.6% and 74.1%, respectively, with a 3-m CEP, but even the JASSM with a significantly smaller explosive yield is more than 95% if a CEP of 2 m can be confidently established.

It is vital to stress that as the accuracy improves, the silo-housed missile is more likely to be destroyed by a direct impact to the silo cover or the silo's wall upon a cruise missile penetrating into it. This is simply a consequence of more impacts closer to the silo. Even partial penetration of the cover followed by a detonation would likely destroy a missile with the shrapnel produced if full penetration and collapse of the silo's cover could

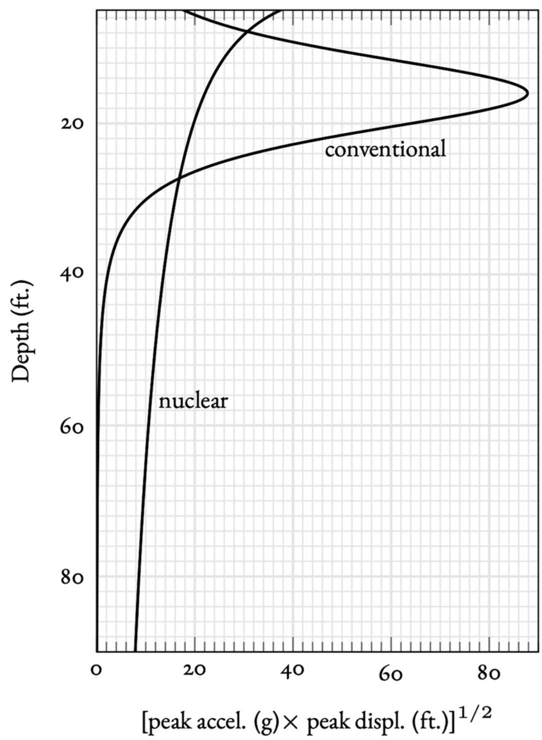


**Figure 10.** The quantity [peak accel. × peak displ.]$^{1/2}$ plotted as it varies with depth for a conventional weapon of 360-lb yield (equivalent to a U.S. Joint Air-to-Surface Standoff [JASSM] cruise missile) detonated 16 ft underground and 9 ft away from a 1,500-psi silo along with the same quantity for a nuclear weapon needed to destroy the missile housed inside this silo. The value plotted represents the ground motion just outside the wall of the silo.

**Table 4.** The SSKP for U.S. Tomahawk and JASSM cruise missiles against a 1,500-psi silo for CEP values of 2 and 3 m.

| Silo hardness = 1,500 psi—Russia and China | | | |
|---|---|---|---|
| Weapon system | Lethal radius (m) | SSKP (CEP = 3 m) | SSKP (CEP = 2 m) |
| Tomahawk | 8.93 | 0.998 | 1.00 |
| JASSM | 5.61 | 0.921 | 0.997 |

*Note.* A 1,500-psi silo is believed to be the hardest strategic missile silo in Russia or China. CEP = circular error probable; SSKP = single-shot kill probability.

**Table 5.** The SSKP for U.S. Tomahawk and JASSM cruise missiles against a 2,000-psi silo for CEP values of 2 and 3 m.

| Silo hardness = 2,000 psi—United States | | | |
|---|---|---|---|
| Weapon system | Lethal radius (m) | SSKP (CEP = 3 m) | SSKP (CEP = 2 m) |
| Tomahawk | 6.95 | 0.976 | 1.00 |
| JASSM | 4.19 | 0.741 | 0.952 |

*Note.* A U.S. Minuteman III ICBM silo may be hardened up to 2,000 psi. CEP = circular error probable; SSKP = single-shot kill probability.

not be assumed. Some countermeasures may complicate destroying the missile by these methods while increasing the chances of destruction with others that use explosive effects (e.g., covering the silo cover with soil or anything that improves the coupling between the explosion and the ground). The point is that all of these would have a very high chance of success with more accurate missiles because successfully piercing the silo cover or crushing the silo's walls with the explosion's effects is not necessary. Yet any countermeasure that can effectively slow down the incoming missile could complicate the success of a direct impact. Importantly, any such passive countermeasure could prove equally effective against a nuclear detonation by breaking up the nuclear airblast wave that must induce the ground motions lethal to a silo. This is discussed further in the section "Silo vulnerability to a direct impact" in the Appendix.

If a direct hit on a silo cover ever becomes too challenging, Figure 11 displays that lethal directly induced ground motions provide an area that is roughly 8 times greater compared to that of a silo cover for a Russian or Chinese 1,500-psi silo and about 13.5 times greater compared to the smaller silo cover of a U.S. 2,000-psi silo. The result is that it may be more advantageous to purposely miss the silo cover should the silo be considered less vulnerable to a direct impact than from a penetrating weapon detonating alongside it. This is not to say that countermeasures covering the area of lethality in Figure 11 are not possible, as anything that can successfully prevent the missile from penetrating into the ground may be effective in protecting the siloed missile. But a more effective defense may be more challenging across a larger area when combined with other missile penetration aids or the involvement of multiple detonations. This is likely an area that could benefit from more research, or at least from bringing together the relevant elements for this specific problem into a coherent presentation.

## Modeling parameter sensitivities

The parameter ranges from Table 3 can be factored into Equation 15 and a range of conventional SSKPs calculated. It deserves mention that the strongest support remains for Equation 15 as it exists, but there are many assumptions made in this analysis, and it is worth testing how sensitive is it to plausible ranges of them.

Equation 15 can first be constructed to maximize conventional lethality. This essentially means that smaller values for nuclear-induced horizontal motions will lower the threshold that conventional weapons must meet to destroy a siloed-based missile. The silo peak velocity should then remain at one (100% of the ground motion's peak velocity), the CEP set to 2 m, and the penetration depth set to optimal (Equation 14). Of course, as discussed earlier, maximum conventional lethality may in fact result from a detonation near the location most relevant for transmitting the maximum acceleration to the missile, but it is unknown where exactly that might be, so maximizing lethality through Equation 15 is the best tool available. Following through here results in the following expression:

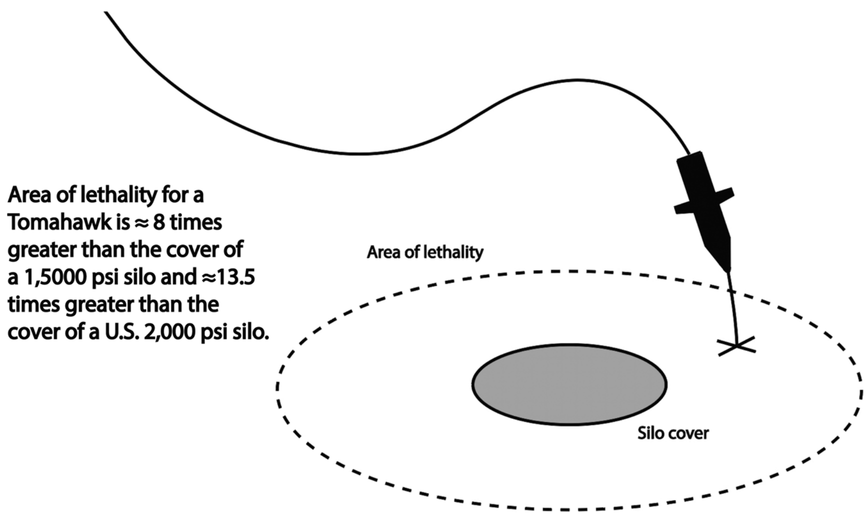


**Figure 11.** The area of lethality for directly induced ground motions due to an underground conventional explosion is represented by the dashed line and that of a direct hit on a silo cover by the sold, gray circle. This dashed area presents a larger targeting area should there be doubts about the success of an attempted direct hit.

$$\overset{\text{Nuclear}}{\left(\frac{2}{3}\right)\cdot\int_{5}^{90}0.109\cdot P_{\mathrm{o}}^{0.89}\cdot Y_{\mathrm{nuc}}^{\gamma}\cdot z^{-0.415}e^{-0.00425(z)}dz} = \overset{\text{Maximum Conventional}}{\int_{5}^{90}158.1\cdot Y_{\mathrm{conv}}^{\frac{5}{6}}\cdot x(x+(z-h)^2)^{-\frac{7}{4}}dz} \tag{17}$$

where the factor of 2/3 on the nuclear side arises from $[2\cdot(1/3)\cdot(2/3)]^{1/2}$ in the expression [peak accel. × peak displ.]$^{1/2}$. The 2 here is needed to cancel the $(1/2)\cdot$ vertical displacement = horizontal displacement originally embedded in Equation 15. The maximum SSKPs calculated here appear in Tables 6 and 7.

When relying on Table 3 to adjust Equation 15 to minimize conventional lethality, a factor $(4/3)^{1/2}$ was added to the nuclear side arising from $[2\cdot(2/3)]^{1/2}$ in the expression [peak accel. × peak displ.]$^{1/2}$. A factor of 1/2 was added to the conventional side as the lower limit of what fraction of the ground's conventional peak velocity would be reached by the silo as it responded to the shorter pulse duration of the conventional ground shock. In addition, the CEP was set equal to 4 m in the SSKP calculation in Equation 16 in case 3 proves too ambitious a capability. Adjusting Equation 15 to minimize conventional lethality provides the following model:

$$\begin{aligned} &\text{Nuclear} \\ &\left(\frac{4}{3}\right)^{\frac{1}{2}}\cdot\int_{5}^{90}0.109\cdot P_{\mathrm{o}}^{0.89}\cdot Y_{\mathrm{nuc}}^{\gamma}\cdot z^{-0.415}e^{-0.00425(z)}dz \\ &\text{Minimum Conventional} \\ &=\left(\frac{1}{2}\right)\cdot\int_{5}^{90}158.1\cdot Y_{\mathrm{conv}}^{\frac{5}{6}}\cdot x(x+(z-h)^2)^{-\frac{7}{4}}dz \end{aligned} \tag{18}$$

Tables 6 and 7 display a broad range of minimum to maximum SSKPs, with the most sensitive parameter being the CEP. If it was 3 m and not 4, a capability the U.S. government still claims it outperforms, the JASSM's minimum respective SSKPs of 0.59 and 0.35 for 1,500 and 2,000 psi would become 0.80 and 0.54, and those for the Tomahawk 0.94 and 0.79. This is with all other parameters chosen to minimize conventional lethality. This section simply adds more useful parameters to the model provided by Equation 15 should specific knowledge about them ever become more firmly established, either in general or in a more specific context.

## Nuclear lethality

An accurate comparison with the lethality of nuclear weapons must adjust the silo hardness to the attacking warhead's explosive yield. With the silo hardnesses based on a 1-Mt yield, the lower yields that accompany nuclear warheads today require that ground motions compensate in order to maintain the same lethality. This is done by adjusting the silo hardness upward, which only has a practical consequence of shortening the lethal radius in order to accurately model how a lower yield remains as lethal to a silo.

The nuclear side of Equation 15 can be integrated over the length of the silo and the average obtained for 1,000-, 1,500-, and 2,000-psi silos (just as was done for Figure 9). This same equation can then be plotted as a function of silo hardnesses for relevant explosive yields of U.S. and Russian ICBM and SLBM warheads. This plot is displayed in Figure 12. The x-value where these lines intersect with the average of $[\text{peak accel.} \times \text{peak displ.}]^{1/2}$ will provide the adjusted silo hardness from either 1,000-, 1,500-, or 2,000-psi levels. Warhead yields of 90 and 455 kt correspond to those on U.S. Trident D5 SLBMs,[112] 300 kt to that on a U.S. Minuteman III ICBM,[113] and 800 kt to that on a Russian SS-18 ICBM.[114] Chinese nuclear warheads were not included only because with yields between 200 and 300 kt,[115] they fit within the U.S. nuclear weapon capabilities included here, albeit with fewer missiles currently and accuracies highly unlikely to approach those of the United States.

A simple lethality comparison for nuclear weapons against 1,500- and 2,000-psi silos were calculated with the same SSKP expression (Equation 16) and displayed in Table 8. Equation 3 was used to calculate the lethal

**Table 6.** The minimum and maximum SSKPs for the U.S. Tomahawk and JASSM cruise missiles against a Russian or Chinese 1,500-psi silo.

| Silo hardness = 1,500 psi—Russia and China | |
|---|---|
| Weapon system | Min–Max SSKP |
| Tomahawk | 0.80–1.00 |
| JASSM | 0.59–0.99 |

*Note*. The parameters from Table 3 were used to construct Equations 17 and 18, the minimum and maximum expressions for conventional lethality. The CEP used for the minimum SSKP here was 4 m, with the maximum SSKP using a CEP of 2 m. CEP = circular error probable; SSKP = single-shot kill probability.

**Table 7.** The minimum and maximum SSKPs for the U.S. Tomahawk and JASSM cruise missiles against a U.S. 2,000-psi silo.

| Silo hardness = 2,000 psi—United States | |
|---|---|
| Weapon system | Min–Max SSKP |
| Tomahawk | 0.59–1.00 |
| JASSM | 0.35–0.98 |

*Note*. The parameters from Table 3 were used to construct Equations 17 and 18, the minimum and maximum expressions for conventional lethality. The CEP used for the minimum SSKP here was 4 m, with the maximum SSKP using a CEP of 2 m. CEP = circular error probable; SSKP = single-shot kill probability.

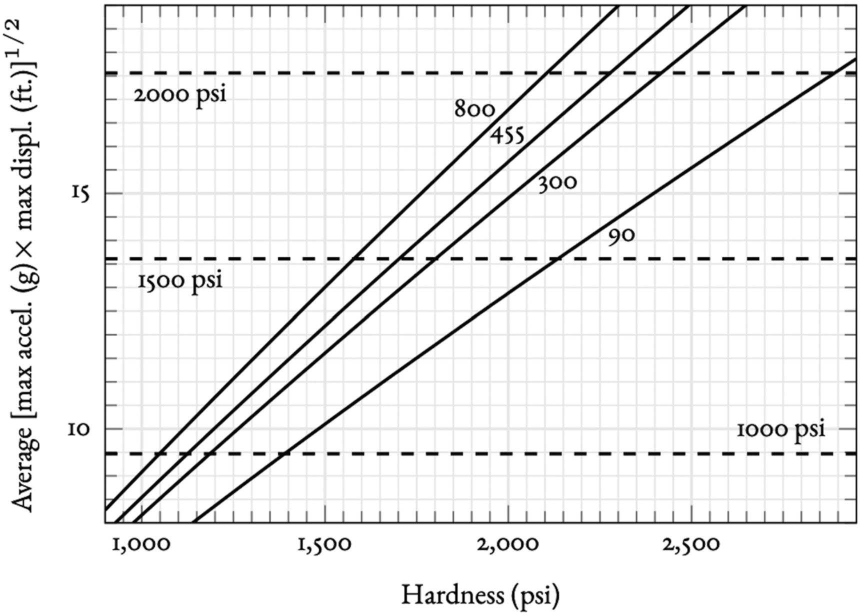


**Figure 12.** The average of the quantity [peak accel. × peak displ.]$^{1/2}$ plotted as a function of silo hardness in psi for warhead yields of 90, 300, 455, and 800 kt. The silo hardnesses will be adjusted upward to a new value provided by the intersection of the warhead's yield with the baseline hardnesses of 1,000-, 1,500-, and 2,000-psi indicated by the dashed lines.

radius, LR, corresponding to the adjusted silo hardness and explosive nuclear yield against these silos for the U.S. and Russian missiles and warheads listed in Table 8.[116] The lethality of the United States' strategic missiles against its own 2,000-psi silos may not be an accurate measure for Russian or Chinese capabilities, but it presents some idea of what the most accurate nuclear ballistic missiles can do. It is not known publicly whether the 120-m CEP listed in Table 8 for the Russian SS-18 reflects current Russian (or Chinese) capabilities. Regardless, such accuracies should be expected to exist on at least some Russian and Chinese missiles.

The important result is that when comparing the calculations in Table 8 with those in Tables 4 and 5, Tomahawk cruise missiles with accuracies of 2 or 3 m are as lethal as U.S. strategic nuclear warheads against U.S. silos, with JASSMs approaching that same lethality at 2-m accuracy. Against Russian or Chinese silos, JASSMs with 3-m accuracy are nearly as lethal. It should be noted that the the SSKPs against 1,500- and 2,000-psi silos for the Tomahawk with 3-m accuracy in Tables 4 and 5 exceed those for the 90 kt W76-1 warhead on a Trident D5 SLBM—probably the most common nuclear warhead in the United States' nuclear war plan.

While it remains doubtful that Russia or China currently possess enough such missiles with the capabilities, or the launch platforms, to threaten the entire U.S. ICBM force, especially considering the missions these weapons would already be assigned closer to their respective homelands, it should be

**Table 8.** The SSKP of a U.S. Minuteman III ICBM W87 warhead and two trident SLBM warheads (W76-1 and W88) against 1,500- and 2,000-psi silos.

| Weapon system | Yield (kt) | SSKP (1,500 psi) | SSKP (2,000 psi) |
|---|---|---|---|
| U.S. Minuteman III ICBM W87 (CEP = 120 m) | 300 | 0.96 | 0.92 |
| U.S. Trident D5 SLBM W76-1 (CEP = 90 m) | 90 | 0.89 | 0.84 |
| U.S. Trident D5 SLBM W88 (CEP = 90 m) | 455 | 1.00 | 0.99 |
| Russian SS-18 ICBM (CEP = 120 m) | 800 | 1.00 | 0.99 |

*Note*. The SSKP for an 800-kt warhead accompanying a Russian SS-18 ICBM is included as well. CEP = circular error probable; SSKP = single-shot kill probability.

anticipated that they will eventually. There is no reason not to anticipate their acquisition of this capability given the desire of many states beyond the nuclear powers to acquire missiles with increased ranges and greater accuracies. The global trend is obvious.

## Discussion

This analysis is the first attempt at assessing the lethality of conventional weapons against strategic missile silos using a comparison with nuclear lethality. The framework provided could be applied to any conventional warhead delivered by gravity bomb, drone, or cruise, ballistic, or hypersonic missile. While the assumptions made to arrive at the result are solidly supported, further research could provide more confirmation.

First, a finite element analysis should verify that a silo's rigid-body conventional response reaches the free-field peak particle velocity in nearly the same time as in the nuclear case. The average of the quantity $[\text{peak accel.}, \times \text{peak displ.}]^{1/2}$ along the length of the silo would then be considered an adequate comparison between the two motions. More detailed knowledge of the shock isolation system would also allow the response of its key element that transmits a lethal force to the missile to be compared. If this element is closer to a conventional detonation due to available penetration capabilities, it could greatly enhance conventional weapons lethality. This is likely the case given how the silo's rigid-body response is supposed to follow free-field ground motions. This possibility should also be viewed within the range of possible upgrades to conventional weapon capabilities beyond those considered in this analysis that may enhance silo counterforce performance.

Plausible uncertainties in other parameters that could lower the SSKPs were also considered, with the dominant sensitivity coming from a missile's CEP. Yet if U.S. government claims are true that CEPs on its precision cruise missiles are less than 3 m, this offers further support to this result. Another affected parameter is that nuclear-induced horizontal motion was likely a bit overstated throughout this analysis: first, in the nuclear lethality model constructed from the 1957 Frenchman Flat test data at the NTS and, second, with the fraction of vertical motion assumed to be horizontal. The motions used were chosen because it was not clear how to accurately adjust them from the most common levels selected in the literature.

Of further significance is that direct hits on the silo cover will become more common as missile accuracy improves (discussed in the section "Silo vulnerability to a direct impact" in the Appendix). Any countermeasure that is able to successfully slow down an incoming missile without coupling an explosion to the silo or ground may successfully protect the siloed missile if this prevents a partial penetration followed by a detonation spraying shrapnel onto it. Such countermeasures may also be effective protection against nuclear detonations by breaking up the nuclear airblast responsible for inducing ground motions, leaving nuclear silo counterforce no more lethal than conventional in this respect. Multiple detonations may also successfully destroy such countermeasures, stressing again that the nature of a strategic conflict without nuclear weapons could evolve far differently than one with them. Further research could investigate this more fully, including the possibility of purposely missing a silo cover given the area of lethality is roughly 8 to more than 13 times larger for a Tomahawk (see Figure 11). Countermeasures that slow down the incoming missile in this area could also adversely affect lethality depending on how deep it penetrates.

There also remains the question of air defense and guidance. While it appears that U.S. cruise missiles remain quite accurate without GPS systems (one source claimed terrain-matching was preferred over GPS), the exact performance without it remains unclear. What is clear is that the U.S. Navy claims missions can be planned for the latest Block V Tomahawk using terrain-matching (TERCOM/DSMAC) without GPS and that the U.S. Air Force claims JASSMs have systems designed to resist GPS jamming or spoofing. These weapons may still possess CEPs less than 3 m with these options, but this is not clear. Some similar uncertainty remains with the capabilities of air defenses against cruise missiles, where no such system can be 100% effective. U.S. missiles may be unable to be reliably detected, raising the question of their stability implications. Any suggestion that a subsonic cruise missile has less counterforce potential or is less of a stability concern compared with a nuclear ballistic missile or some other conventional hypersonic delivery system needs to be supported with how they are detected. This would include detection within the specific military context of a war, as a bolt from the blue attack is a far more remote possibility compared to an escalating conventional conflict. Of course this circumstance would also include a greater number of missiles attacking ICBM silos, which may be far less likely to go undetected. If further research on the relevant questions for air defense is possible, it could prove useful. But this area may be too contingent on the details and evolution of a military conflict to draw any conclusions. This topic is discussed in the section "Missile guidance and air defense" in the Appendix.

While further research in these areas could help clarify some relevant questions, the implications of conventional silo lethality go well beyond the scope of this paper. Perhaps the most important is that broader elements of the current and evolving military context must now be taken into account regarding strategic stability and arms control, including those encompassing the regional defense dynamics in and surrounding Europe and Asia. This paper provides a technical basis for such analyses or possible initiatives.

## Conclusion

This analysis provides a framework for determining the lethality of a conventional weapon of any explosive yield or accuracy against a strategic missile silo of any hardness. It was used to find that the SSKPs against U.S., Russian, and Chinese silos for two U.S. long-range precision conventional cruise missiles are comparable to those for U.S. and Russian nuclear ballistic missiles: typically well above 90%.

This result suggests that long-range conventional weapons may be substituted for the silo targeting roles of nuclear weapons, allowing strategic counterforce capabilities to remain unaffected with far fewer deployed. It also reveals that conventional weapons must now be considered to properly analyze the relative survivability and reliance on specific nuclear forces among nuclear powers that factor into strategic stability concerns and determine nuclear force requirements.

## Notes

1. The term counterforce refers to an attack on targets of military value, which in a nuclear war includes ICBM silos, air bases stationing nuclear-armed bombers, ports for strategic submarines, or installations vital for nuclear command, control, and communications. This is contrasted with countervalue targets, typically meaning an adversary's cities.
2. Conventional weapons refer to those whose explosive energy comes from the chemical energy released when electrons orbiting an atom's nucleus are rearranged by the breaking of chemical bonds held together by the electromagnetic force. By contrast, the energy of a nuclear explosion originates from the splitting apart of protons and neutrons in an atom's nucleus held together by the strong nuclear force. The energy stored in a nuclear bond is on the order of one mega-electron volt (MeV), whereas that for a chemical bond is one electron volt (eV), i.e., a million times less.
3. The Department of Defense defines precision-guided munitions as "[a] guided weapon intended to destroy a point target and minimize collateral damage," as cited by John R. Hoehn, "Precision-Guided Munitions: Background and Issues for Congress," Congressional Research Service, R45996, June 11, 2021, https://sgp.fas.org/crs/weapons/R45996.pdf; The words accuracy and precision typically have different meanings. Precision is a measure of how close repeated measurements are from one another (or in this case, how close repeated shots on a target are from one another). Accuracy consists of both how close a repeated set of measurements are to the true value *and* their precision. If there is no systematic bias consistent for all measurements, accuracy and precision may be used interchangeably. In this paper, the target location is assumed to be known without any other sources of bias, allowing for interchangeable use.

4. Military analysts in Russia specifically mentioned U.S. conventional weapons threats against their silos in 1999 following NATO airstrikes in what was then Yugoslavia. See David Hoffman, "NATO's Campaign Underscores Russia's Military Collapse," *The Washington Post*, June 12, 1999, https://www.washingtonpost.com/wp-srv/inatl/longterm/balkans/stories/russia061299.htm. Near the end of William Clinton's second term as president of the United States, then–Deputy Secretary of State Strobe Talbott discovered in meetings with Russian military and civilian experts that their calculations suggested the United States would have the offensive capability to knock out 90% of Russia's strategic arsenal using its nuclear forces and precision conventional weapons that they had seen on display in Iraq, Yugoslavia, and Afghanistan. See Strobe Talbott, "Unfinished Business: Russia and Missile Defense under Clinton," *Arms Control Today*, June 2002, https://www.armscontrol.org/act/2002-06/features/unfinished-business-russia-and-missile-defense-under-clinton.
5. Richard Boudreaux, "Russia Says Next U.S. Arms Talks Must Include Others," *The Wall Street Journal*, January 14, 2011, https://www.wsj.com/articles/SB10001424052748704307404576079953654840710.
6. Roger N. McDermott and Tor Bukkvoll, "Russia in the Precision-Strike Regime: Military Theory, Procurement and Operation Impact," *FFI Norwegian Defence Research Establishment*, 17/00979, 2017, https://www.ffi.no/en/publications-archive/russia-in-the-precision-strike-regime-military-theory-procurement-and-operational-impact.
7. James C. O'Halloran, ed., "*C-602 (HN-1/-2/-3/YJ-62/X-600/DH-10/CJ-10/HN-2000)*," *IHS Jane's Weapons: Strategic 2015–2016* (United Kingdom: IHS, 2015), 115–19.
8. Joby Warrick, "China Is Building More Than 100 New Missile Silos in Its Western Desert, Analysts Say," *The Washington Post*, June 30, 2021.
9. Sources for Tomahawk: John Keller, "Raytheon to Start Full-Scale Development of Bunker-Busting Tomahawk Missile with Penetrating Warhead," *Military & Aerospace Electronics*, March 5, 2020, https://www.militaryaerospace.com/communications/article/14169216/bunker-busting-tomahawk-missile-warhead, and United States Navy, "Department of Defense Fiscal Year (FY) 2020 Budget Estimates: Navy, Justification Book Volume 5 of 5, Research, Development, Test & Evaluation, Navy, Budget Activity 7," March 2019, Volume 5-251, p. 7 of 58, https://www.secnav.navy.mil/fmc/fmb/Pages/Fiscal-Year-2020.aspx. Source for JASSM: "AGM-158 Joint Air to Surface Standoff Missile (JASSM)," Dyess Air Force Base, Official United States Air Force Website, https://www.dyess.af.mil/Fact-Sheets/Display/Article/267602/agm-158-joint-air-to-surface-standoff-missile-jassm/.
10. The U.S. government has disclosed that the circular error probable (CEP) accuracy of precision-guided munitions is less than 3 m. See Hoehn, "Precision-Guided Munitions," June 11, 2021.
11. For a map displaying a few plausible launch locations of U.S. Tomahawk cruise missiles against Russia's strategic nuclear targets, see Yevgeny Miasnikov, "Precison-Guided Conventional Weapons," in *Nuclear Reset: Arms Reduction and Nonproliferation*, edited by Alexey Arbatov, Vladimir Dvorkin, and Natalia Bubnova (Moscow: Carnegie Moscow Center, 2012), 448, Figure 3, https://carnegieendowment.org/research/2011/03/nuclear-reset-arms-reduction-and-nonproliferation. This map does not include the possible launch locations for executing a nonnuclear strategic attack with JASSMs, which today would appear so complex that no useful map can likely be constructed.
12. The Russian subsonic variants of the Tomahawk and JASSM are respectively the Kalibr and Kh-101; land-based varieties of such weapons could easily be incorporated here, with the scale of future deployments unknown but unlikely to rival

air- and sea-based systems.

13. For a list of current and planned long-range precision capabilities in Europe, see Camille Grand, "Missiles, Deterrence, and Arms Control: Options for a New Era in Europe," *The International Institute for Strategic Studies*, September 2023, Tables 1–2, https://www.iiss.org/globalassets/media-library—content–migration/files/research-papers/2023/09/mdi-european-deterrence/missiles-deterrence-and-arms-control–options-for-a-new-era-in-europe.pdf.
14. Technically a hypersonic weapon is defined as one that flies faster than five times the speed of sound (Mach 5), which includes most ballistic missiles and certainly all such missiles with intercontinental range, but a hypersonic weapon is generally understood as one that flies both faster than Mach 5 and in a non-ballistic trajectory. Hypersonic boost-glide vehicles and cruise missiles are typically referred to as "hypersonic weapons," whereas older, more familiar systems like ballistic missiles are not.
15. Previously planned intercontinental range U.S. hypersonic boost-glided payload delivery vehicles were expected to only contain 68 to 91 kg of conventional high explosive, but planned conventional boost-glide weapons today will have shorter ranges and may have higher yields. See "Final Environmental Assessment for Conventional Strike Missile Demonstration," Space and Missile Systems Center (SMC), U.S. Air Force, August 2010, 9, Table 2-1, https://apps.dtic.mil/sti/pdfs/ADA544112.pdf.
16. Dmitriy Akhmerov, Yevgeniy Akhmerov, and Marat Valeyev, "It Cannot Be Done Quickly. The Might of Nonnuclear Cruise Missiles Is Illusory," *VPK Voyenno-Promyshlennyy Kuryer*, October 2015. The English translation was provided to the author in 2016; https://topwar.ru/84773-po-bystromu-ne-poluchitsya.html.
17. For the purpose of orientation, a silo is a hollow cylinder oriented vertically and extending down into the Earth. It is composed of some mixture of concrete and steel that is designed to protect the missile housed inside from the effects of a nuclear explosion. Each silo has a door that is flush with the surface and typically composed of armored steel that opens, allowing the missile to launch.
18. Harold L. Brode, "Review of Nuclear Weapons Effects," *Annual Review of Nuclear Science* 18 (1968): 180.
19. The two most prominent research articles that developed standard silo vulnerability calculations from this basis were Lynn Davis and Warner Schilling, "All You Ever Wanted to Know about MIRV and ICBM Calculations but Were Not Cleared to Ask," *Journal of Conflict Resolution* 17, no. 2 (June 1973): 207–42, and Kosta Tsipis, "Physics and Calculus of Countercity and Counterforce Nuclear Attacks," *Science* 187, no. 4175 (February 1975): 393–97. Another influential article that came slightly later and discussed more intricate details of the vulnerability problem while also relying on this simple model and Davis and Schilling's calculations was by John D. Steinbruner and Thomas M. Garwin: "Strategic Vulnerability: The Balance Between Prudence and Paranoia," *International Security* 1, no. 1 (Summer 1976): 138–81.
20. This formula is derived from a normal, two-dimensional Gaussian probability distribution

$$P(r) = \frac{1}{2\pi\sigma} e^{-r^2/2\sigma^2}$$

where $r$ is the distance away from the explosion and $\sigma$ is the standard deviation. Integrating this equation from 0 to the lethal radius (LR) and using the definition that 50% of shots will land inside the CEP to eliminate $\sigma$ (by separately integrating from 0 to CEP) allows this simple expression for SSKP to be obtained.
21. This formula is simply converted from the formula for LR in nautical miles on page

213 in Davis and Schilling's "All You Ever Wanted to Know." Missile accuracies (CEPs) in the 1970s were typically expressed in nautical miles, which made such units the standard when discussing nuclear weapons. Today CEPs are measured in meters, providing a rationale for switching units.

22. See Davis and Schilling, "All You Ever Wanted to Know," 213–14.
23. For an even more transparent derivation of LR but with a slightly more cumbersome result, see Tsipis, "Physics and Calculus," 395–96.
24. One example of a graph that displayed how peak overpressure varied over distance from a 1-kt surface burst appears in Samuel Glasstone and Phillip J. Dolan, *The Effects of Nuclear Weapons*, 3rd ed. (United States, 1977), doi:10.2172/6852629. There was also a "Bomb Damage Effect Computer" manufactured by the RAND Corporation and circular slide rule called the "Missile Effectiveness Calculator" manufactured by the Heavy Military Electronics Systems Division of General Electric. All three of these sources were discussed in Davis and Schilling, "All You Ever Wanted to Know."
25. The role of U.S. nuclear weapons has been restated in many U.S. government reports and has three objectives: (1) deter nuclear or all forms of strategic attack; (2) assure allies and partners; and (3) achieve presidential objectives if deterrence fails. All three objectives are said to require that the nuclear targets in adversary countries be held at risk of being destroyed by U.S. nuclear weapons to limit damage to the United States and its allies in the event of a nuclear war. See U.S. Department of Defense, *National Defense Strategy of the United States of America*, https://media.defense.gov/2022/Oct/27/2003103845/-1/-1/1/2022-NATIONAL-DEFENSE-STRATEGY-NPR-MDR.PDF (2022). For a discussion on this strategy and the nuclear posture that underpins it, see Ryan Snyder, "Pushing Theory Beyond the Brink," *New Perspectives* 28, no. 1 (2020): 112–17.
26. Paul H. Nitze, "Assuring Strategic Stability in an Era of Détente," *Foreign Affairs*, 54, no. 2 (January 1976).
27. Steinbruner and Garwin, "Strategic Vulnerability."
28. Pavel Podvig, "The Window of Vulnerability That Wasn't: Soviet Military Buildup in the 1970s—A Research Note," *International Security*, 33, no. 1 (Summer 2008): 118–38.
29. This is discussed in the Defense Intelligence Agency's *Physical Vulnerability Handbook—Nuclear Weapons* (U) (AP550-1-2-60-INT) and its *Mathematical Background and Programming Aids for the Physical Vulnerability System for Nuclear Weapons* (DI-550-27-74), as cited by Steinbruner and Garwin, "Strategic Vulnerability," 142–43, fn 3.
30. The DIA used what is called a vulnerability number (VN) that reflected the target's hardness relative to some damage level, a letter indicating the target's sensitivity to overpressure (P) or dynamic pressure (Q), and a K factor. According to the DIA's *Mathematical Background and Programming Aids for the Physical Vulnerability System for Nuclear Weapons*: "The increased blast duration associated with larger yields may cause targets to fail at lower pressure levels, while at small yields the reduced blast duration may necessitate higher pressures for target failure. To account for this yield dependence, the PV system uses K-factors for both P and Q targets. The K-factor is an integer from 0 to 9 which adjusts the base VN to reflect the sensitivity of the target to the different pressure-time pulse shapes for yields other than 20 kt. A K factor of 0 indicates a target that is not sensitive to blast wave duration and can be expected to fail at the same pressure regardless of weapon yield. A K factor of 9 indicates a target that is very sensitive to blast wave duration and can be expected to fail at quite different pressures at various yields." See *Mathematical Background and Programming Aids for the Physical Vulnerability System for Nuclear Weapons*,

Defense Intelligence Agency, October 1, 1974, 34–38; The VN for a Russian ICBM silo (carrying SS-11/19 ICBMs) was reported as "55L8," where the "L" indicates a target whose probability to severe damage from peak overpressure decreases more rapidly with distance from ground zero compared to P-type silos. The K factor of "8" indicates a target sensitive to the blast wave's duration, suggesting that it would fail at different overpressures for different yields greater than 20 kt. This sensitivity is assumed to apply to other strategic missile silos. This PV number was obtained from the *NATO Target Data Inventory Handbook, 1989*. See Matthew G. McKinzie, Thomas B. Cochran, Robert S. Norris, and William M. Arkin, "The U.S. Nuclear War Plan: A Time for Change," *Natural Resources Defense Council*, (June 2001): 35, Table 3.6, https://www.nrdc.org/sites/default/files/us-nuclear-war-plan-report.pdf.

31. Steinbruner and Garwin, "Strategic Vulnerability," 143, fn 3.
32. Ibid., 143, fn 3.
33. Davis and Schilling, "All You Ever Wanted to Know," 216.
34. The models by which this conclusion is reached are found in Chengqing Wu and Hong Hao, "Modeling of Simultaneous Ground Shock and Airblast Pressure on Nearby Structures from Surface Explosions," *International Journal of Impact Engineering* 31 (2005): 699–717.
35. The U.S. government has only reported that the accuracy (CEP) of its precision-guided munitions are less than 3 m. See IHS Janes "GBU-31/32/38 Joint Direct Attack Munition (JDAM)," June 18, 2019, https://janes.ihs.com/Janes/Display/jalw3667-jalw, as cited by Hoehn, "Precision-Guided Munitions," June 11, 2021. This 3-m CEP is almost certainly not the correct accuracy measurement for precision-guided cruise missiles. Such missiles likely have more close misses of targets than the Gaussian distribution underlying the CEP measure indicates; these missiles today are also terminally guided by relying on some image of the target to match that detected by a missile. It is unclear how the accuracy is affected, but it likely departs from the standard CEP distribution.
36. Carl H. Builder, "Strategic Conflict Without Nuclear Weapons," RAND Corporation, April 1983, 22.
37. The paper stated that Russian Major General Vladimir Belous asserted in 2009 that an ICBM silo could only be disabled by the penetration of the armored roof of a silo. It is not clear whether the authors meant that a cruise missile would have to fully penetrate the silo cover or whether partial penetration accompanied by the explosive charge's detonation would suffice. The paper seemed to suggest that the Russian Academy of Sciences agreed, with too many conventional precision systems required to destroy Russia's Strategic Nuclear Forces and the required number being even higher if the cruise missiles' guidance systems could be jammed. These claims do not account for all of the mechanisms by which silos may be destroyed (including even by direct impact) nor is it clear on what the claims about guidance systems or air defense penetration are based. The paper also seems to only consider the rarest military context in which this would occur, that being a bolt from the blue attack rather than arising from the evolution of a military conflict. See Akhmerov, Akhmerov, and Valeyev, "It Cannot Be Done Quickly," October 2015. The English translation was provided to the author in 2016. The argument made here was that in the case in which U.S. cruise missiles had CEPs of 3 m (the most accurate CEP the authors considered), this would only provide an SSKP of 50% against a silo door with a radius of 3 m. According to Equation 4, five missiles would then need to be launched at each silo to raise the total kill probability to more than 95%—a typical standard necessary to conclude with high confidence that the intended target will

be destroyed. This, of course, leaves aside the question of missile reliability, which cannot be presumed to be 100%.

38. Vladimir Dvorkin, "Preserving Strategic Stability Amid U.S.-Russian Confrontation," Carnegie Moscow Center, Carnegie Endowment for International Peace (February 2019), https://carnegiemoscow.org/2019/02/08/preserving-strategic-stability-amid-u.s.-russian-confrontation-pub-78319.
39. The SSKP of a missile with a 2-m CEP capable of destroying a missile inside a silo with a 3-m radius in this fashion would be about 79%. With two missiles, it would be 95%. For a missile with a 1-m CEP, the SSKP would be nearly 100%.
40. U.S. Congress, House Committee on Armed Services, Hearings on Military Posture, 97th Cong., 2nd sess., 1981, 85.
41. Hobson, "Minuteman/MX System: Becoming Vulnerable?," 125; Kosta Tsipis, *Arsenal: Understanding Weapons in the Nuclear Age* (New York: Simon & Schuster, 1983): 137.
42. Art Hobson, "Minuteman/MX System: Becoming Vulnerable?," in *The Future of Land-Based Strategic Missiles*, ed. Barbara G. Levi, Mark Sakitt, and Art Hobson (New York: American Institute of Physics, 1989), 125.
43. U.S. Congress, House Committee on Armed Services, Hearings on Military Posture, 97th Cong., 2nd sess., 1981, 85.
44. Office of Technology Assessment, *MX Missile Basing* (September 1981).
45. This disclosure of a 2,000-psi limit for current U.S. silos came from the Air Force, as reported by the Surveys and Investigative Staff of the House Committee on Appropriations. It was stated that decreased hardness from less rattlespace due to a larger MX missile would be compensated for with an upgraded shock isolation system. The report went on to state that the Department of Defense in Fiscal Year 1984 Congressional Hearings requested $450 million to study superhardness recommended by the Scowcroft Commission. It was estimated that due to encouraging developments in superhardening technology, a "hardness of 26,000 psi was attainable without significant design sophistication." The Air Force later notified Congress that current silos would have to be completely demolished and rebuilt or new silos constructed at other locations if R&D determined that superhardness was required. In particular, the rattlespace would have to be increased for additional hardness against nuclear bursts. Given that the diameters of U.S., Russian, and Chinese silos all seem to have comparable rattlespaces and considering that Russian and Chinese ICBMs have larger diameters compared to the U.S. Minuteman IIIs (see section on "Silo hardness in the United States, Russia, and China" herein), the question of why silos with larger rattlespaces have not been constructed in any of these countries arises. This is particularly relevant as China is constructing new silos with similar dimensions to those in Russia. Cost could certainly be a factor, but it may also be that increased hardness is not easily attainable by simply increasing the available rattlespace. See U.S. Congress, House Committee on Appropriations, Military Construction Appropriations for 1985, 98th Cong., 2nd sess., Part 5, 1984, 1010.
46. Lauren Caston, Robert S. Leonard, Christopher A. Mouton, Chad J. R. Ohlandt, Craig Moore, Raymond E. Conley, and Glenn Buchan, "The Future of the U.S. Intercontinental Ballistic Missile Force," RAND Corporation, 2014, 34, Figure 3.4, https://www.rand.org/pubs/monographs/MG1210.html.
47. The compressive strength of concrete can easily exceed 2,000 psi and is usually between 4,000 and 5,000 psi in several warehouses, factories, or for other uses that support heavy loads. Compressive strength does not depend on blast duration or the total delivered overall impulse in this context, so the fact that hardening above

peak overpressure is easily possible is relevant for concluding that the structural integrity of silos will remain intact at a level of 2,000 psi. See GRA-Rock, "Understanding Concrete Strength: From PSI to Tips for Pouring Concrete," December 8, 2020, https://www.gra-rock.com/post/understanding-concrete-strength.

48. Several publications in the 1970s and 80s explicitly stated that psi hardness levels were based on a 1-Mt yield, but John Steinbruner and Thomas Garwin discussed this 1-Mt basis for the silo overpressure proxy model after examining the unclassified portions of the DIA's *Physical Vulnerability Handbook—Nuclear Weapons* (AP550-1-2-60-INT). Some mathematical excerpts of this handbook appear in the DIA's *Mathematical Background and Programming Aids for the Physical Vulnerability System for Nuclear Weapons*, but there is no discussion here of the 1-Mt basis for silo hardness. Steinbruner and Garwin simply state that for yields of 1 Mt and smaller, the effect of using the overpressure proxy model is to understate the hardness of American missile silos. This suggests this model does not take into account blast durations shorter than that accompanying a 1-Mt explosion, indicating its basis for the proxy model's psi hardness numbers. A 1986 report from the Congressional Budget Office on the Trident SLBM also mentioned that its reference yield for its peak overpressure calculations was 1 Mt. See The Congress of the United States Congressional Budget Office, "Trident II Missiles: Capability, Costs, and Alternatives," July 1986, https://www.cbo.gov/sites/default/files/99th-congress-1985-1986/reports/doc18b-entire_0.pdf.
49. Steinbruner and Garwin, "Strategic Vulnerability," 142–43, fn 3.
50. General Electric Company–TEMPO, *Nuclear Geoplosics Sourcebook, Volume IV, Part I, Empirical Analysis of Ground Motion from Above and Underground Explosions*, March 1979, IV-1-1.
51. A seismic wave refers to a mechanical wave of acoustic energy that propagates through the Earth or along its surface. There are generally two types: body and surface waves. Body waves travel through the Earth and are composed of compressional waves and shear waves. Compressional waves produce a force into the Earth along its direction of propagation, whereas shear waves produce one perpendicular to the propagation direction. Compressional waves are effectively the seismic velocity, as they will travel faster than shear waves and will also travel faster than the peak stress front. There is not enough space to present a full primer on the different features of explosion-induced ground motion, but these features do affect key parameters related to silo vulnerability.
52. Brode, "Review of Nuclear Weapons Effects," 191–92.
53. There has been debate whether any silo could survive the fracturing and excavation of the rock or soil around it due to cratering, but the 2,000-psi overpressure threshold for U.S. silos means they will not survive a fair distance outside of a crater in any case.
54. General Electric Company–TEMPO, *Nuclear Geoplosics Sourcebook*, IV-1-2.
55. Ibid., IV-1-2.
56. U.S. Congress, House Committee on Armed Services, Hearings on Military Posture, 97th Cong., 2nd sess., 1981, 85.
57. "The levels from 1/2 to 2 kb correspond to the onset of gross rock failures for most formations and thus represent the range of survival for the best examples of underground construction" (see Brode, "Review of Nuclear Weapons Effects," 200). With the smallest direct–induced motion displayed in Figure 4 being 1/2 kilobar at roughly 230 m near the surface, it should be expected that such motion will almost completely disappear by 300 m. No direct–induced motion would then be expected

to contribute at ranges of interest significantly beyond that.

58. Brode, "Review of Nuclear Weapons Effects."
59. Eugene Sevin, "Validation Testing of Nuclear Survivable Systems" (Proceedings of 59th Shock and Vibration Symposium Volume IV, Technical Report No. SAND88-2473C, Albuquerque, NM, Sandia National Laboratories, December 1988), https://apps.dtic.mil/sti/pdfs/ADA214581.pdf.
60. See Glasstone and Dolan, *The Effects of Nuclear Weapons*, 255, Figure 6.27a. The radius of the inner lip of a nuclear crater produced by a surface burst in hard rock is roughly calculated according to the basis of such a crater formed by a 1-kt surface explosion, which is 50 ft according to Figure 6.27a on page 255. This radius is then scaled up by the weapon yield in kilotons raised to the 0.3 power. So for a 1-Mt (1,000 kt) explosion in hard rock, the radius of the inner lip of a crater produced would be $50 \times (1{,}000)^{0.3} = 397$ ft, or 121 m. This appears close to that displayed for a surface burst crater in Figure 4 for the crater's inner lip radius at ground level (crossing 0 m depth). For smaller explosive yields, the radii of the craters and direct–induced stress contours would shrink accordingly.
61. It is also possible that some shock-absorbing materials could be added outside the silo's external walls to dampen the ground motion as a way to limit the resulting motion of the silo.
62. Eugene Sevin, "ICBM Modernization: A Shock and Vibration Perspective," Keynote speech delivered to the 57th Shock & Vibration Symposium, October 13–16, 1986, New Orleans, LA, *Shock and Vibration Bulletin* (Washington, D.C.: Naval Research Laboratory, 1987).
63. Superhard silos were supposed to protect an enclosed missile from a nuclear airblast at ranges greater than the inner bowl radius of a crater. They needed to survive peak overpressures of around 7,500 atmospheres (~110,000 psi), according to Department of Defense comments. It is not possible to determine whether this hardness referred to protecting the silo structure or the missile itself protected by a shock isolation system. Regardless of what design limits were discovered or accepted, superhard silos were never built and the silo hardness limit became 2,000 psi. See John Michener, "Design and Feasibility of Superhard Silos," *Science & Global Security* no. 2 (1991): 197–98, doi:10.1080/08929889108426360.
64. U.S. Congress, House Committee on Appropriations, Military Construction Appropriations for 1985, 98th Cong., 2nd sess., 1984, 619.
65. Walter Pincus and Martin Schram, "More Tests on Silos Needed for Proof MX Plan Will Work," *The Washington Post*, December 4, 1982, https://www.washingtonpost.com/archive/politics/1982/12/04/more-tests-on-silos-needed-for-proof-mx-plan-will-work/263f8359-bba7-4f6c-b9e3-d108c2e58d21/.
66. The Minuteman III ICBM today is suspended in a cage within a silo that is not too dissimilar from an MX canister. See David K. Stumpf, "Ballistic Missile Shock Isolation Systems," *Air & Space Power History* 69, no. 4 (Winter 2022): 31–42.
67. Zhou Anfeng et al., "Anti-Explosion and Shock-Absorbing Design of Bomb-Canister System in Launch Silo," *Journal of National University of Defense Technology* 45, no. 1 (February 2023): 102–09. doi:10.11887/j.cn.202301011. The author translated the Chinese text to English with Google Translate.
68. James C. O'Halloran, ed., "DF-5," *IHS Jane's Weapons: Strategic 2015–2016* (United Kingdom: IHS, 2015), 7–8.
69. Miko Vranic, "Russia's Sarmat Super-Heavy ICBM Undergoes First Full Flight Test," Janes, April 21, 2022, https://www.janes.com/defence-news/news-detail/russias-sarmat-super-heavy-icbm-undergoes-first-full-flight-test.

70. "R-36 (SS-18 "Satan")," Missile Threat, CSIS Missile Defense Project, Center for Strategic and International Studies, last updated August 2, 2021, https://missilethreat.csis.org/missile/ss-18/.
71. Eugene Miasnikov, "Counterforce Capabilities of Conventional Strategic Arms" (presentation at ISODARCO XXVI Winter Course "New Military Technologies: Implications for Strategy and Arms Control, January 6–13, 2013, Andalo, Italy, slide 14), https://www.armscontrol.ru/pubs/en/Miasnikov-PGM-130110.pdf.
72. Hans Kristensen, "China's Expanding Missile Training Area: More Silos, Tunnels, and Support Facilities," Federation of American Scientists, February 2, 2021, https://fas.org/publication/plarf-jilantai-expansion/.
73. Hans M. Kristensen, Matt Korda, Eliana Johns, and Mackenzie Knight. "Russian Nuclear Weapons, 2024," *Bulletin of the Atomic Scientists* 80, no. 2 (2024): 118–45, doi:10.1080/00963402.2024.2314437.
74. Akhmerov, Akhmerov, and Valeyev, "It Cannot Be Done Quickly," October 2015.
75. These data come from the archival collection of Vitali Kataev at the Hoover Institution Archive at Stanford University. See Pavel Podvig, "The Window of Vulnerability That Wasn't," *International Security* 20, no. 1 (Summer 2008): 131, Table 3.
76. U.S. Congress, House Committee on Appropriations, Military Construction Appropriations for 1985, 98th Cong., 2nd sess., 1984, 1010.
77. Podvig, "The Window of Vulnerability That Wasn't," 131.
78. Stanley Wilson and Earl Sibley developed what appear to be the most accepted methods of obtaining the displacement and particle velocity–time histories of ground motion (see Stanley D. Wilson and Earl A. Sibley, "Ground Displacements from Air-Blast Loading," *Proceedings of the ASCE*, no. 88, SM6 [December 1962]: 1–31) but they apparently do not provide as accurate a calculation as correlation measurements discussed in Sections IV-1.2 and IV-1.3 of General Electric Company–TEMPO, *Nuclear Geoplosics Sourcebook*, IV-1-105.
79. James L. Drake and Charles D. Little, Jr., *Ground Shock from Penetrating Conventional Weapons* (Vicksburg, MS: Army Engineer Waterways Experiment Station, May 1983).
80. Robert J. Odello and Paul Pierce, "Ground Shock Effect from Accidental Explosions" (Technical Report 4995, Department of the Army, Picatinny Arsenal, November 1976).
81. Drake and Little, *Ground Shock from Penetrating Conventional Weapons*, 3.
82. Ibid., 3.
83. Shashank Pathak and G. V. Ramana, "Air-Blast Induced Ground Displacement," *Procedia Engineering* 173 (2017): 555–62.
84. Drake and Little, *Ground Shock from Penetrating Conventional Weapons*, 4.
85. Ibid., Figure 3, 4. Data taken from Table 5.1 in P. S. Bulson, *Explosive Loading of Engineering Structures* (London: E&FN SPON, 1997), Chapter 5.
86. This expression does not appear to differ much for other geologies relevant for silo locations.
87. C. W. Lampson, "Effects of impact and explosions," *Explosions in Earth* (Washington, D.C.: NRDC, 1946): Vol. 1, Chapter 3, as cited in Bulson, *Explosive Loading of Engineering Structures*, Chapter 2.
88. A cruise missile will fly farther the less it weighs. But better penetration capability depends on a penetrator's length and density, which works against the goal of a longer-range missile. Warheads on cruise missiles with penetration capability are thus long, thin, and dense to enhance their penetrative power while limiting the weight they add to the cruise missile. A JASSM has a WDU-42/B warhead (other-

wise known as a J-1000), while a Block V Tomahawk has a WDU-43/B. Both are penetrators, with the WDU-43/B on the Tomahawk containing a shaped charge. The Tomahawk's current warhead may not be designated WDU-43/B anymore, but the current one still has common penetrating features. A J-1000 warhead has a length of 72 inches, and a 2005 National Academy of Sciences study on earth-penetrating weapons stated that an optimized penetrator could have a cross-sectional density exceeding 25,000 $kg/m^2$. At this cross-sectional density, the JASSM's J-1000 warhead would have an overall density of 13,700 $kg/m^3$, and only a density of 8,000 $kg/m^3$ is necessary for a 6-foot-long warhead to penetrate four times its length to 24 ft in reinforced concrete. With the optimal penetration depth for the Tomahawk being 23 ft, it appears capable of reaching that depth if equipped with a similar J-1000 warhead, as appears likely. On maximum penetration depth, see Robert W. Nelson, "Low-Yield Earth-Penetrating Nuclear Weapons," *Science & Global Security* 10 (2002): 1–20, https://scienceandglobalsecurity.org/archive/2002/01/low-yield_earth-penetrating_nu.html; for optimized penetrator cross-section density, see National Research Council, *Effects of Nuclear Earth-Penetrator and Other Weapons* (Washington, DC: The National Academies Press, 2005), doi:10.17226/11282; and for details about the J-1000 warhead, see John W. Jones. 2004. Energy Dense Explosive. US Patent 6,679,960 B2, filed 21 April 2001, and issued January 20, 2004, https://patents.google.com/patent/US6679960B2/.

89. Reaching a depth of four times the warhead's length assumes a terminal velocity for a penetrating warhead of ~1000 m/s (Mach 2.9). There is every reason to believe that the Tomahawk and JASSM possess this capability, but this information is not available publicly.

90. Information about the TNT equivalents for common explosive fill materials range from 1.3 for Tritonal to 1.65 in AFX-757, the known high explosive in the JASSM. With these TNT equivalences constantly changing, a compromise was made to scale every conventional high explosive's weight up by a factor of 1.5. Some materials may be slightly higher or lower than this, but too much focus here is a distraction. With the explosive material for the JASSM currently at 1.65, using 1.5 was viewed as a conservative value. See Stefan K. Kolev and Tsvetomir T. Tsonev, "Aluminized Enhanced Blast Explosive Based on Polysiloxane Binder," *Propellants, Explosives, Pyrotechnics* 47, no. 2 (February 2022).

91. A ratio of horizontal to vertical ground compressions $C_h/C_v$ reflecting what fraction of the vertical displacement is horizontal is given by

$$\tan\alpha = \frac{C_h}{C_v} \tag{19}$$

where

$$\alpha = \arcsin\frac{f(M_c)}{U} \tag{20}$$

with $M_c$ the constrained modulus of deformation, $U$ the velocity of the air blast, and $f$ some component of $M_c$ reflecting its magnitude at the shock front. See Wilson and Sibley, "Ground Displacements from Air-Blast Loading," 26–28.

92. The ratio H/V of horizontal to vertical motions induced by the compression wave is equal to

$$\tan\beta = \frac{H}{V} \tag{21}$$

where

$$\beta = \arcsin\frac{C_p}{U} \quad (22)$$

with $C_p$ the compressional wave's velocity and $U$ the velocity of the air blast. See General Electric Company–TEMPO, *Nuclear Geoplosics Sourcebook*, IV-1-44–IV-1-48.

93. Wilson and Sibley, "Ground Displacements from Air-Blast Loading," 23, Figure 14.
94. Ibid., 28.
95. Brode, "Review of Nuclear Weapons Effects," 190.
96. U.S. Congress, House Committee on Appropriations, Military Construction Appropriations for 1985, 98th Cong., 2nd sess., 1984, 619.
97. Duncan Lennox, "LGM-118 Peacekeeper," in *Jane's Strategic Weapon Systems* (London: IHS Janes, 2011).
98. "Strategic Forces: Minuteman Weapons System Status and Current Issues," *United States General Accounting Office*, September 1990, 13, https://www.gao.gov/assets/nsiad-90-242.pdf.
99. Taking an average would suggest dividing the integrals in Equation 15 by the silo's length. These integrals were divided by 90 ft, the length of a U.S. Minuteman ICBM silo, but it should probably have been 85 given the integral was taken from 5 to 90. But the values obtain here and plotted on the y-axis of Figure 9 do not matter; it is the values on the x-axis that will determine the appropriate detonation point for a conventional weapon.
100. As covered in the section "Airblast-, direct-, and crater-induced ground motion," nuclear ground motion in this analysis is dominated by airblast-induced motion, whereas the conventional motions are induced by direct effects.
101. The original model appeared in J. H. Haywood, "Response of an Elastic Cylindrical Shell to a Pressure Pulse," *Quarterly Journal of Mechanics and Applied Mathematics* 11, Part 2 (1958): 129–41, and provided the basis for the modification in F. Finlayson, L. E. Fugelso, and Y. Shulman, "Design Procedures for Shock Isolation Systems of Underground Protective Structures Volume II: Structure Interior Motions Due to Directly Transmitted Ground Shock" (General American Transportation Corporation, Technical Report No. RTD-TDR-63-3096, Air Force Weapons Laboratory, Kirkland Air Force Base, New Mexico, December 1965), https://apps.dtic.mil/sti/citations/AD0627597.
102. This conclusion was true for step, rectangular, and exponentially decaying pressure pulses. See F. Finlayson, L. E. Fugelso, and Y. Shulman, "Design Procedures for Shock Isolation Systems of Underground Protective Structures Volume II: Structure Interior Motions Due to Directly Transmitted Ground Shock" (General American Transportation Corporation, Technical Report No. RTD-TDR-63-3096, Air Force Weapons Laboratory, Kirkland Air Force Base, New Mexico, December 1965), https://apps.dtic.mil/sti/citations/AD0627597.
103. It is important to state that Figure 8 represents the responses for a step pulse incident on the cylinder, but those for an exponentially decaying pulse were said to be similar.
104. Travel time is the ratio of the time elapsed after the incidence of the pulse to that needed for the pulse to travel the length of the cylinder's radius at the speed of the seismic velocity. See H. Huang, "An Exact Analysis of the Transient Interaction of Acoustic Plane Waves with a Cylindrical Elastic Shell," *Journal of Applied Mechanics* 37, no. 4 (December 1970): 1091–99.
105. Drake and Little in *Ground Shock from Penetrating Conventional Weapons* provided the expression for a exponentially decaying pressure pulse and stated that the pulse would decay to nearly zero in 1 to 3 travel times: a travel time defined as the dis-

tance from the detonation divided by the seismic velocity. For a JASSM cruise missile, one travel time is approximately 10 ft away (i.e., the distance of the detonation from the silo), or roughly a missile silo's radius.

106. Because the conventional side of Equation 15 does not have a simple antiderivative, numerical approximations were made of this integral to compare with the nuclear side. Several such approximations were made for all of the conventional yields in Figure 9 across a set of ranges that would intersect with the 1,000-, 1,500-, and 2,000-psi nuclear values. And then a separate fit to this data was made for each yield.
107. United States Navy, "Tomahawk Cruise Missile," last updated September 27, 2021, https://www.navy.mil/Resources/Fact-Files/Display-FactFiles/Article/2169229/tomahawk-cruise-missile/.
108. Gareth Jennings, "USAF Launches JASSM New Variant Missile," *Janes*, October 25, 2022, https://www.janes.com/defence-news/news-detail/usaf-launches-jassm-new-variant-missile.
109. "AGM-158 Joint Air to Surface Standoff Missile (JASSM)," Dyess Air Force Base, Official United States Air Force Website.
110. A U.S. ICBM silo is approximately 12 ft in diameter and 90 ft long. See United States General Accounting Office ,"Strategic Forces: Minuteman Weapons System Status and Current Issues," September 1990, 13, https://www.gao.gov/assets/nsiad-90-242.pdf.
111. Hoehn, "Precision-Guided Munitions," June 11, 2021.
112. Hans M. Kristensen, Matt Korda, Eliana Johns, and Mackenzie Knight. "United States Nuclear Weapons, 2024," *Bulletin of the Atomic Scientists* 80, no. 3 (2024): 182–208. doi:10.1080/00963402.2024.2339170.
113. Ibid., 183, Table 1.
114. Hans M. Kristensen, Matt Korda, Eliana Johns, and Mackenzie Knight. "Russian Nuclear Weapons, 2024." *Bulletin of the Atomic Scientists* 80, no. 2 (2024): 118–45. doi:10.1080/00963402.2024.2314437.
115. Hans M. Kristensen, Matt Korda, Eliana Johns, and Mackenzie Knight. "Chinese Nuclear Weapons, 2024." *Bulletin of the Atomic Scientists* 80, no. 1 (2024): 49–72. doi:10.1080/00963402.2023.2295206.
116. The respective accuracies of a Minuteman III ICBM and Trident D5 SLBM are believed to be 120 and 90 m. See James C. O'Halloran, ed., "LGM-30G Minuteman III," in *IHS Jane's Weapons: Strategic 2015–2016* (United Kingdom: IHS, 2015): 105–7; James C. O'Halloran, ed., "*UGM-133 TRIDENT D-5,*" *IHS Jane's Weapons: Strategic 2015–2016* (United Kingdom: IHS, 2015): 112–14.
117. Some may argue that penetrating the silo's wall sideways at some depth followed by a detonation within it would be another option. This may be correct, but the prospect of penetrating into the wall at some depth seems more uncertain than the main destruction mechanism considered in this paper. The two mechanisms are almost equivalent, in that it is the silo's motion and not damage to its structure that is lethal to the missile, but there is less uncertainty with reaching the optimal depth first and then using explosion-induced ground motions.
118. Katie Walker, "Shaped Charges Pierce the Toughest Targets," *Science & Technology Review*, June 1998, https://str.llnl.gov/content/pages/past-issues-pdfs/1998.06.pdf.
119. Eugene Miasnikov, "Counterforce Capabilities of Conventional Strategic Arms," January 6–13, 2013.
120. Akhmerov, Akhmerov, and Valeyev, "It Cannot Be Done Quickly," October 2015.
121. The optimal limit of ~1,000 m/s can be found in Nelson, "Low-Yield Earth-Penetrating Nuclear Weapons," 7, Figure 2, and the limit of 1,200 m/s can be found in Nancy F. Swinford and Dean A. Kudlick, *A Hard and Deeply Buried Target Defeat*

*Concept* (Sunnyvale, CA: Lockheed Martin Missiles & Space, 1996), 2, https://apps.dtic.mil/sti/pdfs/ADA318768.pdf. The Swinford and Kudlick paper suggests that for impacts greater than 4,000 ft/s (~1,200 m/s), the penetrator risks failure from case erosion.

122. For the Chinese YJ-18 anti-ship cruise missile, see Jane's Navy International, "Ship Killers: New Anti-Ship Cruise Missiles Raise the Stakes in Northeast Asia," April 2018. For the Russian Kalibr anti-ship cruise missile, see OE Data Integration Network, "3M54K(SS-N-27 Sizzler) Russian Medium-Range Anti-Ship Cruise Missile," https://odin.tradoc.army.mil/exports/pdf/05f6b7cded0d8dc47eaf55bc57fcc93dd0f73350.pdf.
123. If resilience to erosion of a warhead penetrator case can be improved at greater speeds, this judgment may need to be changed. But this may only help hypersonics compare more favorably with subsonics if the former have difficulty slowing down in the terminal phase of flight. It may enhance lethality of both, however, if subsonic penetrators armed with explosives are then more effective against possible passive countermeasures designed to slow down incoming missiles.
124. Yangyu Lu, Qingming Zhang, Yijiang Xue, Cheng Shang, Wenjin Liu, Siyuan Ren, and Renrong Long, "Hypervelocity Impact Cratering on Semi-Infinite Concrete Targets of Projectiles with Different Length to Diameter Ratios," *Applied Sciences* 10, no. 11 (2020): 3910, doi:10.3390/app10113910.
125. Richard H. Speier, George Nacouzi, and K. Scott McMahon, *Cruise Missile Penaid Nonproliferation: Hindering the Spread of Countermeasures Against Cruise Missile Defenses* (Santa Monica, CA: RAND Corporation, 2014).
126. United States Navy, "Department of Defense Fiscal Year (FY) 2020 Budget Estimates," March 2019, Volume 5-250, p. 6 of 58.
127. Geoffrey B. Irani and James P. Christ, "Image Processing for Tomahawk Scene Matching," *Johns Hopkins APL Technical Digest* 15, no. 3 (1994), https://secwww.jhuapl.edu/techdigest/Content/techdigest/pdf/V15-N03/15-03-Irani.pdf.
128. Frederick W. Riedel, Shannon M. Hall, Jeffrey D. Barton, James P. Christ, Brian K. Funk, Thomas D. Milnes, Peter E. Neperud, and David R. Stark, "Guidance and Navigation in the Global Engagement Department," *Johns Hopkins APL Technical Digest* 29, no. 2 (2010), https://secwww.jhuapl.edu/techdigest/Content/techdigest/pdf/V29-N02/20-02-Riedel.pdf.
129. United States Navy, "Department of Defense Fiscal Year (FY) 2020 Budget Estimates," March 2019, Volume 5-250, p. 6 of 58.
130. John Grady, "Entire Navy Tomahawk Missile Arsenal Will Upgrade to Block V," *USNI News*, U.S. Naval Institute, January 22, 2020, https://news.usni.org/2020/01/22/entire-navy-tomahawk-missile-arsenal-will-upgrade-to-block-v.
131. United States Air Force, "Department of Defense Fiscal Year (FY) 2019 Budget Estimates: Air Force, Justification Book Volume 3a of 3, Research, Development, Test & Evaluation, Air Force, Vol–III Part 1," February 2018, Volume 3a-514, p. 4 of 10, https://www.saffm.hq.af.mil/FM-Resources/Budget/Air-Force-Presidents-Budget-FY19/.
132. "Lockheed Martin to Build GPS-Guided Missiles with Infrared Seeker Technology for Launch from B-2 Bombers," *Military & Aerospace Electronics*, May 23, 2023, https://www.militaryaerospace.com/sensors/article/14294347/infrared-seeker-gps-missiles.
133. George N. Lewis and Theodore A. Postol, "Long-range Nuclear Cruise Missiles and Stability," *Science & Global Security* 3, no. 1–2 (1992): 49–99, doi: 10.1080/08929889208426379.
134. John W. R. Lepingwell, "Soviet Strategic Air Defense and the Stealth Challenge," *International Security* 14, no. 2 (1989): 85. doi:10.1162/isec.14.2.64.
135. Lewis and Postol. "Long-Range Nuclear Cruise Missiles and Stability."

136. “The Main Vulnerability of the Russian S-400 Is Revealed,” *VPK Voyenno-Promyshlennyy Kuryer*, August 2, 2022, https://vpk.name/en/619642_the-main-vulnerability-of-the-russian-s-400-is-revealed.html.
137. Robert Dalsjö, Christofer Berglund, and Michael Jonsson, “Bursting the Bubble? Russian A2/AD in the Baltic Sea Region: Capabilities, Countermeasures, and Implications” (Report No. FOI-R–4651–SE, Swedish Defence Research Agency), March 4, 2019, https://www.foi.se/en/foi/reports/report-summary.html?reportNo=FOI-R–4651–SE.
138. Roger McDermott, “Russian Air Defense and the US Strike on Al-Shayrat,” *Eurasia Daily Monitor*, April 11, 2017, https://jamestown.org/program/russian-air-defense-us-strike-al-shayrat/.
139. Dalsjö, Berglund, and Jonsson, “Bursting the Bubble?,” 48.
140. Office of the Historian, United States Department of State, “The Limited Test Ban Treaty, 1963,” https://history.state.gov/milestones/1961-1968/limited-ban.
141. Atmospheric tests from this height would limit the radioactive fallout when compared to a burst at the surface that would eject large volumes of soil into the atmosphere.
142. Henry F. Cooper, Jr., and Jimmie L. Bratton, *Calculation of Vertical Airblast-Induced Ground Motions from Nuclear Explosions in Frenchman Flat* (Technical Report No. AFWL-TR-73-111, Air Force Weapons Laboratory), October 1973, https://apps.dtic.mil/sti/citations/AD0914362.
143. Ibid., iii/iv.
144. Ibid., iii/iv.
145. See “Literature Cited,” in Brode, “Review of Nuclear Weapons Effects,” 202.
146. Brode, “Review of Nuclear Weapon Effects,” 185.
147. Ibid., 185.
148. Brode mentions that $C_L$ = 4,000 ft/s in concrete and 3,000 ft/s in soil. Brode, “Review of Nuclear Weapons Effects,” 190.
149. Brode claims that acceleration is measured in grams, but this is likely just an error. Farther down on page 190 for ground motions expressions in the outrunning region, Brode says that acceleration is measured in *g*’s. What was meant is almost certainly units of gravity *g*, which is the standard unit for measuring acceleration for ground motion. See Brode, “Review of Nuclear Weapons Effects,” 190.
150. Brode,” Review of Nuclear Weapons Effects,” 190.
151. General Electric Company–TEMPO, *Nuclear Geoplosics Sourcebook.*
152. Cooper and Bratton, *Calculation of Vertical Airblast*, 38.
153. Ibid., 39.
154. Ibid., 49.
155. Ibid., 38.
156. Ibid., 32.
157. Ibid., 34, Figure 20.
158. L. M. Swift, D. C. Sachs, and F. M. Sauer, *Ground Motion Produced by Nuclear Detonations* SRI (Unpublished), as cited by General Electric Company–TEMPO, *Nuclear Geoplosics Sourcebook*, IV-1-48.
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161. Ibid., 105.
162. Stumpf, “Ballistic Missile Shock Isolation Systems,” 31–42.
163. Ibid.

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## Appendix

This appendix discusses additional elements relevant to conventional lethality that need to be considered in more detail and provides a broader technical basis of support to the model used to calculate the single-shot kill probabilities. Topics in the first category include discussions of silo vulnerability to direct impacts, missile guidance, and air defense. Topics in the second include a justification of the nuclear lethality model constructed from analyses of the 1957 Nevada Test Site data that only rises to a peak overpressure of 1,000 psi, a Chinese analysis of shock isolation systems, and a simple model of shock isolation used to derive the key parameters for comparison between nuclear and conventional ground motions.

## Silo vulnerability to a direct impact

As conventional weapons become more accurate, more direct missile impacts with a silo will occur, preventing destruction of the missile inside by the silo's response to explosion-induced ground motions. This explains the need to consider direct silo impacts. In the case of a nuclear explosion, the ground motions induced by a nuclear airblast will destroy the missile anyway, making any consideration of a direct impact superfluous. But such an analysis for conventional weapons becomes more important because instead of penetrating into concrete or soil alongside a silo, they will need to penetrate into the armored steel of a silo cover if they land closer to its center. If they cannot penetrate a silo cover effectively, it is not clear whether the missile could be destroyed by airblast-induced ground motions with a surface or near-surface burst or, alternatively, whether the silo itself could be sufficiently moved before the ground by such a burst.

There appear to be three ways to destroy missiles by direct impact: (1) complete penetration of the silo cover that causes it to crater into the silo's interior; (2) partial penetration combined with a detonation[117]; and (3) a direct impact that rattles that missile inside a silo. This last option appears difficult to model but may be easier to test. A horizontal component would be needed (just like ground motion) in order to destroy the missile, but this method may be the most likely way to ensure this. Figure A1 displays the area of lethality defined by the maximum distance from which ground motions induced by underground conventional explosions can destroy missiles, but the area of a silo's cover is displayed separately to indicate its resistance to this effect if it is directly impacted. The pull-up, pull-down terminal trajectory here allows for more control of the impact velocity and angle at the target.

The full details of penetration (e.g., material properties, impact of velocities and angles of entry with the target) are not considered in this paper. Yet both U.S. cruise missiles emphasized here possess penetration capabilities, so even partial penetration of a few ft into a silo's armored cover combined with a detonation should be considered enough to destroy the remainder of the cover that exists below the point of detonation, spraying heavy shrapnel into the silo and onto the missile as a result. An argument could be made that this is more likely with a shaped charge possessed by the Tomahawk that can arguably penetrate deeper before detonating, but this difference compared to the JASSM does not seem significant. A shaped charge has already been shown to penetrate 3.4 m into high-strength armored steel in tests,[118] and the maximum thickness of a Russian silo cover is 1.8 m,[119] so even reasonable partial penetration here would seem sufficient to blow through a cover when combined with an explosion.

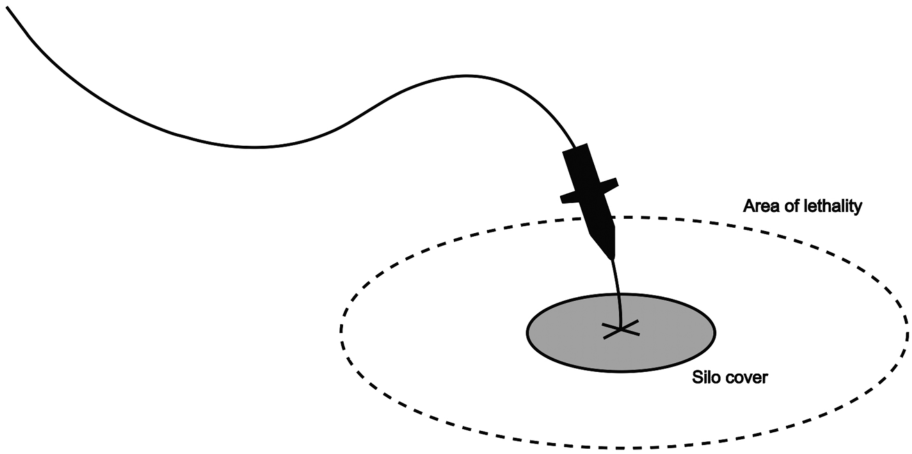


**Figure A1.** The area of lethality defined by its lethal radius, the farthest distance away from a silo that a conventional detonation could destroy the missile inside, and the area of a silo's cover that defines where a missile must land to destroy it by direct impact. The trajectory displayed here is a classic pull-up, pull-down maneuver, which can control the velocity and angle of entry at target impact.

As previously discussed, some interpretation of this was made in an article appearing in a Russian military journal that appeared to reflect an assessment by the Russian Academy of Sciences. The assumption was that penetration with a powerful explosive charge would be needed to "decommission" the silo, with the article assuming that any direct impact would accomplish this.[120] Penetration would take advantage of enhancing the coupling of the explosion to the interior of the armored cover compared with an airblast that occurred at its exterior.

There is an important point to be made with respect to penetration effectiveness between subsonic and hypersonic missiles. That is, for a subsonic cruise missile to penetrate to deeper depths, its terminal velocity must increase relative to its cruising speed, whereas for a hypersonic boost-glide vehicle or cruise missile it must decrease. There is an optimal speed to maximize the depth of penetration, somewhere between ~1,000 and 1,200 m/s depending on the source.[121] Anti-ship varieties of both Russian and Chinese subsonic cruise missiles appear to possess terminal velocities in this range (Mach 2.5–3.0),[122] but it is unclear whether land-attack varieties do. No terminal velocity of a U.S. cruise missile with penetration capability appears to exist in the public domain, but there is little reason to doubt it would depart from that accepted as optimal. Hypersonic missiles then, despite their faster speed (equal to or greater than Mach 5), are not able to destroy targets more effectively with penetration compared with subsonic options and may be less effective if they are unable to slow down sufficiently in their terminal phase or as a consequence of a smaller explosive yield if an attack needs to be combined with a detonation.[123]

One advantage hypersonic weapons may eventually possess, however, is the capability of steel projectiles to crater the cover of a silo by their kinetic energy alone. This appears to be affected by the projectile's length and diameter, and their erosion would not be affected at the same speed of impact as it would for a penetrating warhead case.[124] This prospect would allow their speed to present a lethality advantage over a subsonic cruise missile if this destruction mechanism was eventually considered the most dominant among others utilizing a direct impact.

There may be countermeasures able to slow down incoming missiles complicating direct impact effectiveness. The key for a defender would be to avoid those that improve the ability of detonations to more effectively couple to the ground or enhance their explosive coupling in a way that threatens the silo cover or allows silos to be sufficiently moved to threaten the siloed missile. Any set of strong wires or cages, or really anything that provides an air gap that would not aid in coupling an explosion to the ground or silo cover, may be effective. For instance, a thicker armored silo cover or rocks or topsoil covering it may allow a conventional detonation to bury more effectively and better direct an explosion downward, even though these countermeasures would likely offer better protection from the impact of speed alone. Yet some measures could complicate the ability to launch a siloed missile, perhaps making them undesirable.

Countermeasures involving an air gap designed to slow down an incoming conventional missile may also break up a nuclear airblast as it moves along the ground toward the silo. The total force felt by the ground with these measures would be weakened if it was partly required to impact some implement set above the ground, adversely affecting the strength of the lethal motions needed to sufficiently move the silo. Nuclear counterforce may provide no clear advantages over conventional in this case.

More active point defenses such as a Gatling gun or a blast fragmentation warhead fired into the air may aid in destroying incoming warheads. These may depend on the ability to detect an incoming cruise missile, however, which is not a forgone conclusion, and any active defenses would likely need to contend with some sort of pull-up, pull-down terminal trajectory designed to complicate point defenses. Further questions concern the capability of any active defense to defend a silo against a missile that may attack from all angles with a range of maneuvering capabilities, as well as the range at which an incoming missile is detected. For example, if a Gatling gun is able to successfully hit a Tomahawk cruise missile late in its terminal flight but above its desired aim point, would it matter or would the missile still be able to effectively penetrate the ground and successfully detonate as originally planned?

The first burden for silo point defense would appear to be on successfully detecting a missile an adequate distance away from a silo, but it remains to be demonstrated that this is a reliable prospect. The military context also matters, with the availability of air defenses dependent on the evolution of a military conflict and militaries likely to learn to better penetrate them as conflicts go on. In addition, cruise missiles themselves could be equipped with a variety of penetration aids, further complicating point defense effectiveness.[125] This is a complex area where the factors at play in offense and defense never remain static. This also explains the need for more research. The next section on "Missile guidance and air defense" provides some more expansive arguments related to some of these issues. It should be noted, however, that if it a direct hit cannot be effective, the most successful response may be to miss the silo cover on purpose and land within the area of lethality depicted in Figure A1. Kill probability calculations would then depend on some elements of the relevant guidance systems.

## Missile guidance and air defense

For conventional weapons to effectively destroy a silo-housed missile, their guidance system must remain functioning throughout their flight path and they must penetrate any air defense arrangement to land within the lethality radius surrounding the target. While this topic is beyond the scope of what was investigated in this analysis, it remains crucial to assess in a more comprehensive way. Because while subsonic cruise missiles may be regarded as stabilizing compared to the higher speeds of ballistic and non-ballistic hypersonic missiles, this simple characterization may be inaccurate if they cannot be reliably detected and unless their capabilities are placed into the context of an evolving military i.e., which targets and air defense systems are destroyed in a specific military context before a conflict evolves to the strategic level with the same varieties of cruise missiles. Without supported justifications regarding both detection and the context of war, claims about the more limited or stabilizing counterforce potential of subsonic conventional cruise missiles due to their slower speed simply cannot be considered valid. This is especially true with any comparison to a bolt from the blue attack with nuclear intercontinental range ballistic missiles. Such a comparison fails to account for the relevant permutations of military conflict regarding today's military context that it is not only irrelevant—it is deeply misleading. The circumstances involved may be of an entirely different character than those of a conflict with nuclear weapons.

On the of question of the continuity of the guidance system, the use of GPS in U.S. weapons systems has become so ubiquitous that its disruption by jamming or spoofing is viewed as a significant concern. Tomahawk cruise missiles since the 1970s have relied on terrain contour matching (TERCOM) and digital scene matching area correlator (DSMAC). Both TERCOM and DSMAC have been retained in the latest Block V variant of the Tomahawk as backups in case the GPS becomes unusable.[126] TERCOM matches terrain elevation with a radar altimeter, while DSMAC matches scenes with optical images.

It remains unclear whether GPS disruption would be a significant problem for attacking land targets given the TERCOM and DSMAC capabilities. A 1994 article stated that DSMAC was preferred at that time over GPS "to attack targets most effectively with a conventional warhead."[127] And while TERCOM appears to function better over rough terrain given the altimeter measurements it makes, DSMAC appears quite capable at most points over land.[128] GPS appears required while the Tomahawk flies over water. DSMAC and GPS are mentioned as the typical terminal guidance systems for the Tomahawk, but the exact performance capabilities of one over the other do not appear to be significant. Given the threats to GPS, not relying on it is vital and appears possible. There may be information that demonstrates a clear difference in capabilities revealing GPS guidance to be superior, but this does not appear to be the case. After all, on the Block V Tomahawk, the U.S. Navy's budget document states: "The NAV/COMMs upgrade allow for planned missions using Terrain Contour Matching (TERCOM)/Digital Scene Matching Area Correlation (DSMAC) updates without GPS."[129] The U.S. Navy also would not have acquired the Block V and upgraded all Block IV Tomahawks to the Block V[130] if they were unable to successfully strike their targets without being aided by GPS.

There is no indication that JASSMs are guided by terrain-matching capabilities, but the U.S. Air Force continues to purchase them, in addition to several U.S. allies, indicating that they are able to withstand the challenges with GPS somehow. Air Force budget documents from 2018 on research and development listed an anti-jam/anti-spoofing system as part of a GPS receiver capable of receiving Military Code (M-Code), a communication considered more secure for GPS operation.[131] Follow-on developments with the JASSM-ER suggested continued reliance on GPS and an infrared terminal seeker; an internal navigation system was also mentioned.[132] All signs suggest that the Air Force is confident about its level of GPS reliance for this key weapons system.

Regarding air defense, the question is whether cruise missiles can be reliably detected. It first needs to be stated that whatever radar might be used for this purpose would likely be destroyed before a scenario involving precision strikes against strategic missile silos would arise. This is what makes this so different than typical strategic stability assessments with nuclear weapons. Imagined scenarios in the midst of a war might involve such precision strikes over days, where there may not be a nuclear response, especially if no lives are lost in the process. It is very easy to imagine different national leaders reacting differently here or, for that matter, deciding differently on whether to initiate such an attack. This is of course less likely if the strikes were with nuclear weapons in which millions could be killed. Plus, detecting JASSMs is considered almost impossible due to their sleeker shape that is more advantageous for reflecting radio waves and avoiding detection compared to Tomahawks, but there are several factors to consider.

One is that the United States would not have purchased the Block V Tomahawk if it was not able to reach its target and destroy it. Other options would exist; they would likely just be more expensive. This does not mean that strategic missile silos were the intended targets, but Tomahawks still need to avoid similar means of detection in order to reach their intended targets. Another is that an over-the-horizon (OTH) radar that would initially detect incoming cruise missiles from several hundred or more than 1,000 km away will not operate well against Tomahawks at night (due to a lower required operating frequency) and under certain conditions where a significant echoed clutter creates a background that complicates missile detection.[133] The radar cross section (RCS) of a Tomahawk was known in the 1980s to be roughly 0.1 $m^2$.[134] It could easily be lower today, but at least at operating frequencies where the radar's wavelength exceeded the length of the missile (10–60 m), the cross-section would not matter and certain operating parameters of an OTH radar would instead determine prospects for missile detection. The JASSM may be vulnerable at these very low frequencies, too. These parameters would include the signal/noise ratio and the signal/clutter ratio.[135] It is uncertain how a Russian OTH radar might match up against a Tomahawk, especially against the large number of missiles that would be required to destroy many silos. But it is important to keep in mind two things: (1) an attack against silos would likely occur after cruise missiles had already flown into a country to destroy other targets, leaving it unclear whether the silos were at any point the intended targets; (2) an OTH radar would likely be destroyed in a war by the time this scenario arose. But if more comprehensive research on this is possible, it could illuminate an important aspect of this issue. A small-scale attack at the right time of day would likely go undetected; the question is a whether a large one would too.

Another question is whether silos could be defended as point targets against cruise missile attacks. If cruise missiles are detected by an OTH radar first, a system like a Russian S-400, generally assumed to be more effectively than an American Patriot system,[136] would likely be better prepared to intercept the missiles later in their flight since they know they are coming. If they go undetected, however, how successful might a system like the S-400 system prove to be? This discussion really only applies to the Tomahawk, as the JASSM and JASSM-ER on fifth-generation F-35s would be hard for an S-400 to detect.[137] The S-400 appears to have a range that is only effective to around 40 km, possibly less depending on the terrain, but a U.S. aircraft such as a EW-18G Growler may provide electronic warfare cover to a cruise missile attack, as was done in a 2017 U.S. attack against a Syrian airbase with Tomahawks.[138] This may complicate any attempt to intercept them, although such an aircraft likely could not accompany missiles all the way to a Russian silo field. Yet other electronic countermeasures could almost certainly accompany a Tomahawk (e.g., electronic jamming or wire chaff).[139] It is likely that the best chance an air defense system would have is some advanced warning of an incoming attack in which an exploding warhead or warheads can be fired into the air around a silo as the missile approached. The timing would likely have to be just right and an incoming missile's incoming pull-up, pull down trajectory may create other complications. Early-arriving decoys might also cause such a system to deplete itself or misdirect the fire. Of course further research may uncover more insight, but it appears that a lot must go right to successfully destroy an incoming Tomahawk. Most of the burden here may just be reliably detecting the incoming missile.

## Modeling nuclear lethality

Reinventing the way that nuclear counterforce is calculated for the purpose of assessing its conventional counterpart depends on accurately modeling the ground motions induced by the nuclear airblast moving along the ground away from the point of a nuclear explosion. This is easily the most uncertain element in this effort.

Like so many other real-world physical phenomena, reliance on well-known relationships in physics can be misleading at best when attempting to understand physical behavior. This follows in this case not only from the varying properties of different geologies where relevant missile silos are stationed, but the important details of ground motion are affected by inhomogeneities that are not evident unless detailed studies of the ground are conducted. This makes any general physical relationship between the parameters of a nuclear explosion and ground motion next to impossible. Compounding this challenge is the meager amount of test data limiting the effectiveness of models constructed from ground motion measurements. The Limited Test Ban Treaty signed by the United States, the Soviet Union, and Great Britain in 1963 eliminating nuclear tests in the atmosphere, outer space, and underwater[140] limits the data available in this area because underground tests do not provide the same ground motion geometry relevant for silo vulnerability, where airblast-induced motion would travel down from the surface.

Fortunately a believable argument can be made that a close approximation of ground motion is possible with the data that exists. During the military program Operation Plumbbob in 1957, there were 24 nuclear tests at the Frenchman Flat lake bed at the Nevada Test Site (NTS). One of the 46 projects composing this program (Project 1.5) observed vertical and radial accelerations and vertical displacements produced in the ground from an explosion called Shot Priscilla. This test burst had an estimated yield of 37 kt from a height of 700 ft.[141] A report produced years later in 1973 recounts how laboratory stress-strain data, soil index characteristics, and seismic data were used to synthesize material properties for use in first principal calculations.[142] From there a simple model was constructed of peak vertical particle velocities and displacements as a function of yield, peak overpressure, and depth. Priscilla ground motion data at Frenchman Flat was then compared to this model and found to be "reasonably consistent."[143] The report further mentioned that the model provided "a primary basis of empirical prediction procedures widely used in the design and analysis of strategic structures during the past 10 years."[144] Thus it remains possible that the models in this report were still used several years later to somehow obtain the 2,000-psi silo hardness number reported during the years of the MX missile debate.

To illustrate how ground motion calculations can vary, it is worth mentioning that the expressions included in Brode's paper yield results that are significantly smaller than those calculated from the Frenchman Flat. They were also taken from an earlier version of the *Nuclear Geoplosics, Volume IV* report in 1964 and were absent in the later 1979 version referenced earlier.[145] They still contain some important conceptual knowledge about the physics, however, and provide recommendations related to horizontal ground motion.

Knowing the overpressure impulse $I_p^+$ is relevant because that is related to how much the ground beneath will be displaced. It can be expressed as follows[146]:

$$I_p^+ \simeq 1.83(P_o)^{1/2} Y^{1/3}\left[1+0.00385(P_o)^{1/2}\right] \tag{A1}$$

At the relatively high overpressures (i.e., >1,000 psi) needed to destroy a silo, most of the impulse is delivered in the first few milliseconds, making the duration of the impulse much less important than at lower overpressures.[147] The vertical ground displacement depends on the impulse and its maxima at a depth of 5 ft below the surface is given by

$$d_{\text{vert}} \simeq 20 I_{\text{p}}^{+} (P_{\text{o}})^{1/4} / (SC_{L}) \text{ft} \pm 30\% \tag{A2}$$

where $S$ is the specific gravity ($S = 2$) and $C_L$ is a parameter related to the seismic velocity whose value indicates how effectively the ground shock is loaded onto the ground to move it.[148] This expression contains an interesting bit of physics that empirically determined models lack, which is that the higher the impulse, the larger the ground displacement. It is worth keeping in mind that this dependence remains a feature of ground displacements. Expressions for the vertical acceleration and velocity maxima at 5-ft depth follow:

$$a_{\text{vert}} \simeq 340 P_{\text{o}} / C_{L} \pm 30\% \tag{A3}$$

$$v_{\text{vert}} \simeq 75 P_{\text{o}} / SC_{L} \text{ ft/sec} \pm 30\% \tag{A4}$$

where $a_{\text{vert}}$ is measured in g's.[149]

Of crucial importance is how vertical motion relates to horizontal because the silo-housed missile must experience some acceleration in the horizontal direction if it is to be destroyed by rattling off the silo's inner walls. This must be true due to the stronger horizontal ground motion present on the side of the silo closest to the detonation. Brode recommends a value for horizontal displacement that is roughly half that of vertical displacement, with the horizontal values for velocity and acceleration being about equal to the vertical values.[150] Other sources recommend horizontal displacements that are even less than half of the vertical value and horizontal velocities and accelerations that are less than rather than equal to these vertical values, but these percentages can vary greatly with depth.[151] In the interest of remaining conservative regarding what conventional weapons may effectively destroy, the modeled nuclear ground motions may be greater than they are in reality, thereby forcing conventional weapon–induced motions to match. For this reason, Brode's recommendations will be applied:

$$\text{Horizontal Displacement} = \frac{1}{2}\text{Vertical Displacement}$$
$$\text{Horizontal Velocity \& Acceleration} = \text{Vertical Velocity \& Acceleration}$$

This also has the additional advantage of keeping the models of ground motion simpler than they would be otherwise. Determinining how to apply the relevant angles of the ground shock with the horizontal direction at various depths involves so much uncertainty that the decision was made to keep the relationships simple and increase the burden on conventional weapon performance against silos.

Of further note is that Equations A2 through A4 are calculated for motions 5 ft under the surface. There are likely two reasons this shallow depth was chosen, both of them related. One is that the ground motion near the surface from an airblast-induced effect is simply not predictable. The ground is not very compact near the surface compared to farther below, making any general relationship between input parameters and the physical ground motion calculation unreliable at best. Second, and this is particularly true with horizontal motion, the possibility that a good deal of the ground soil can be ejected into the atmosphere close to the surface as a shockwave passes would likely make a focused horizontal force against the silo less likely at shallower depths. For these reasons, the calculated ground motions at a 5-ft depth are probably more appropriate to start from when considering ground motions capable of moving a silo.

Returning to the nuclear tests conducted at the Frenchman Flat, the 1973 report that provided the model and data comparison offered the following expression for calculating the vertical displacement $d$ (in inches) as a function of depth $z$ (in feet) below the surface:[152]

$$d = 7\left(\frac{P_o}{100\,\text{psi}}\right)^{0.78}\left(\frac{Y}{1\,\text{Mt}}\right)^{\alpha} e^{-0.0085(z)}\ \text{(in.)}\quad \text{for } 0 \le z \le 100\,\text{ft} \tag{A5}$$

where

$$\alpha = 0.15\left(\frac{P_o}{100\,\text{psi}}\right)^{0.16} \tag{A6}$$

The difficulty is that while this expression is considered valid for yields $Y$ between 1 kt and 10 Mt, it is only considered so for overpressures between 100 and 1,000 psi. Thus some further investigation is necessary before claiming it is appropriate to use up to 2,000 psi.

One point is that Equation A5 shows $d$ to be strongly dependent on the peak overpressure $P_o$. There is no reason to believe that this strong dependence between 100 and 1,000 psi will increase at higher $P_o$ levels leading to overestimates of conventional capability. If anything, there would be a diminished dependence from $d \propto P_o^{0.78}$ instead of ground displacement continuing to maintain this proportionality. The ground motions induced by airblasts propagating over the ground are at higher frequencies compared to direct–induced motion, so there will be a lower limit to how much the ground can move in this case. And this displacement will just become more limiting as $P_o$ increases. This is a desirable development for assessing conventional lethality because, as stated earlier, it likely overestimates nuclear-induced ground motion, thereby forcing conventional weapons to meet a more ambitious threshold. A further argument along those lines is that, in general, the attenuation rate of all ground motion increases with depth as $P_o$ increases.[153] Yet Equation A5 displays a rate that is largely independent of this, making these calculations again likely overestimates of nuclear motion.

Of further relevance in deciding whether Equation A5 is credibly applicable is the comparison with data provided in Figures A2 through A5. Figure A2 displays peak displacement data with depth for independent peak overpressure data obtained by the Stanford Research Institute (SRI) and Sandia. But with such limited amounts of data, the accuracy of the model is difficult to assess, given the scatter in the data inherently expected with depth. A somewhat better comparison is provided in Figure A4, which displays the peak vertical displacement data normalized to the best fit of the experimental surface data, shown by the dashed line in Figure A3.[154] Both present an adequate fit to the data, suggesting that—together with the previous arguments regarding the physics of ground motion—Equation A5 should be a valid expression at peak overpressures up to 2,000 psi. This fit to the test data at the Frenchman Flat results in larger ground motions than the models in Brode's 1968 paper, was published later in a 1973 report after a detailed soil property analysis, and appeared to replace Brode's referenced expressions in a later edition of the

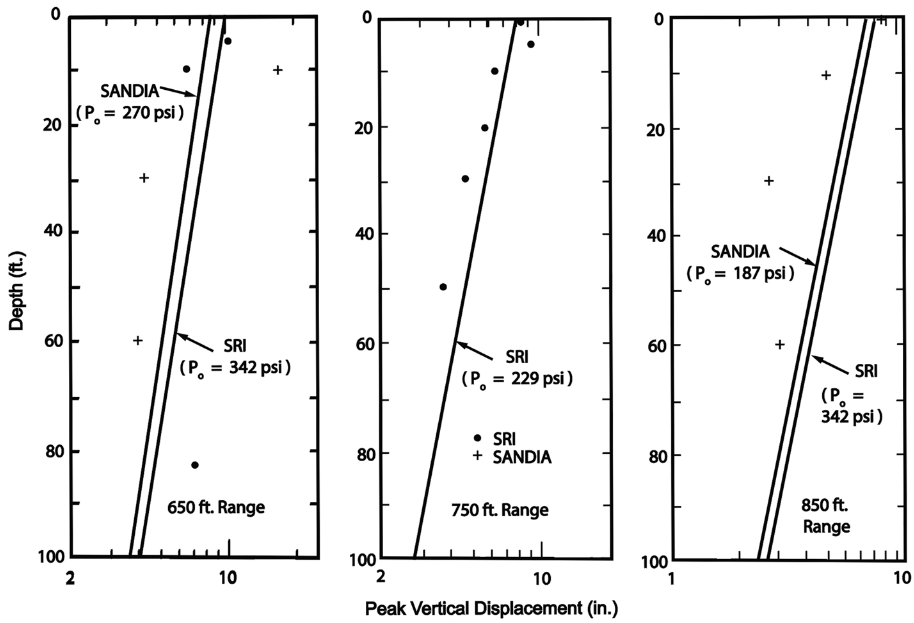


**Figure A2.** How calculated and measured peak vertical displacements (in inches) vary with depth (in feet) at 342, 270, 229, and 187 psi. Modified from "Calculation of Vertical Airblast-Induced Ground Motions from Nuclear Explosions in Frenchman Flat," 45.

*Nuclear Geoplosics Sourcebook, Volume IV*. It also likely overestimates nuclear ground motion, serving to enhance the credibility of conventional weapon capabilities.

For both peak acceleration and velocity, motion near the surface challenges any easy relationship between peak overpressure and these two values. This is consistent with Brode's calculations of these measurements beginning at a depth of 5 ft. In the case of peak velocity, no clear calculation appears reliable at depths shallower than 30 ft, given the expression given in the 1973 report[155]:

$$v = 50\left(\frac{P_o}{100\,\text{psi}}\right)^{0.95}\left(\frac{Y}{1\,\text{Mt}}\right)^{\beta} e^{-0.0085(z-30)}\ (\text{in./sec.})\ \text{for } 30 \le z \le 100\,\text{ft} \qquad \text{(A7)}$$

where

$$\beta = 0.07\left(\frac{P_o}{100\,\text{psi}}\right)^{0.36} \qquad \text{(A8)}$$

There are expressions provided in the report to calculate peak velocity at shallower depths, but the comparison with the data is not as adequate as it is with the displacement data. The peak velocity calculations significantly underestimate this quantity for depths shallower than 30 ft,[156] and one graph only provides some calculations at depths of 0, 10, and 30 ft.[157] None of these methods for determining peak velocity appear reliable. This is not crucial for the ultimate comparison with ground motion resulting from a conventional explosion, however, so leaving peak velocity due to nuclear-induced motion as represented

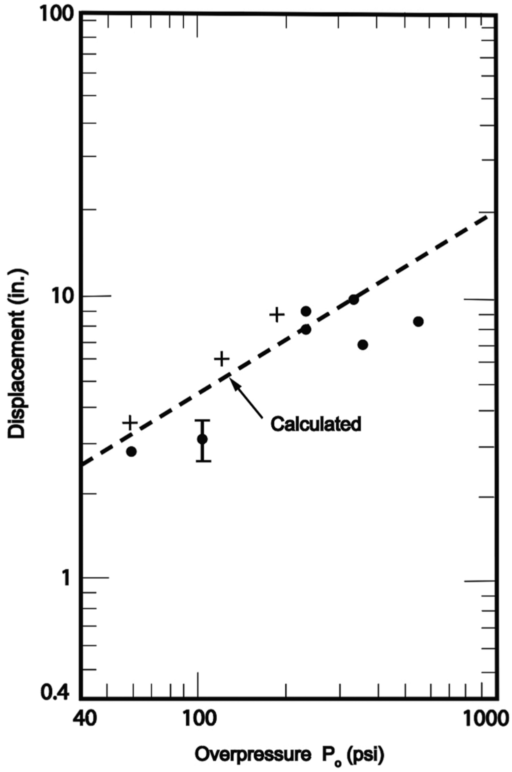


**Figure A3.** Calculated estimate of very near-surface vertical displacement (in inches) data from Priscilla Shot. Modified from "Calculation of Vertical Airblast-Induced Ground Motions from Nuclear Explosions in Frenchman Flat," 50.

by Equations A7 and A8 is fine. Fortunately there is a solid relationship for determining peak vertical acceleration (in units of $g$) with depth $z$:[158]

$$a_v = 1.5 \cdot P_o \cdot z^{-0.83} \ (g) \tag{A9}$$

It is also suggested that the horizontal acceleration is approximately one-third of the vertical near the surface,[159] but accelerations were kept equal between the two directions as discussed earlier.

Uncertainties in nuclear airblast-induced ground motions are apparent here, but attempts were made to overestimate them to err on the side of increasing the performance burden on conventional weapons, thus enhancing their credibility. This appears to have been accomplished for both peak displacement and acceleration, while the uncertainties for peak velocity are inevitably unimportant in the nuclear–conventional comparison.

## A Chinese perspective on shock isolation

A 2023 paper by rocket engineers in China seemed to confirm the use of peak parameter values by using peak displacements and accelerations in a finite element method to model the response of a missile inside a canister to the motions induced on a silo by a 200-kt nuclear ground burst. The goal was to determine which of three shock isolation arrangements (a suspension-type, lower-support, or slant-type system) provides the most survivability to a silo-based missile by limiting the acceleration felt by the missile.[160]

The missile and canister were both modeled as flexible bodies instead of rigid ones, which is necessary for effective shock isolation design. And the stiffnesses of shock absorbers were adjusted to optimize the displacement and acceleration felt by the missile. A stiffer absorber would increase the acceleration felt by the missile while limiting its displacement, while one less stiff would do the opposite. The paper concluded that the larger acceleration responses were generally at the head, middle, and tail of the missile, with the most likely collision between to the missile and launch canister to occur in the middle. Among all three systems modeled, the suspension-type limited accelerations most effectively.[161]

The paper also claimed that horizontal motion is more harmful than vertical in considering how to protect the missile. It should be mentioned that this analysis is for a missile placed inside a canister similar to the retired U.S. MX missile, but there is little reason to doubt this is true for a missile without one. A Minuteman III missile today is even suspended within a cage that is conceptually similar for survivability purposes to the MX's canister.[162] More interesting is the suggestion that vertical motion could be used in the slant-suspended shock isolation system to weaken the more harmful horizontal motions. This seems possible by forcing the horizontal motion vertically at a different frequency to use up energy, although it is unclear how much control would exist over this in the design. But this system was not considered the most desirable for limiting accelerations felt by the missile overall, with the suspension-type and lower-support systems not displaying any vertical–horizontal coupling.[163] It appears for the most effectively designed shock isolation systems, vertical motion is insignificant enough that it can be neglected.

These are valuable insights about how such systems work that do not appear to be publicly available as a result of similar work done in the United States. They are also relevant given China's silo construction, indicating ongoing research on how to better protect silo-based missiles. But most importantly, understanding what is known about shock isolation systems matters because the relevant shock profiles for conventional- and nuclear-induced ground motions lethal to a silo do not neatly overlap (see Figure 7) and greater insight here will strengthen this lethality assessment. It is also valuable whenever possible to show how different countries that are suspicious of each other think similarly about the survivability of their nuclear arsenal.

## A simple model of shock isolation

This section models a comparison between nuclear and conventional ground motion by equating their early shock isolation system responses shortly after the silo is moved by the ground. Beginning with the idea of a simple mass spring system that is driven by a wall, the spring's compression $x$ can be represented by an equation of motion

$$m\frac{\partial^2 x}{\partial^2 t} = F(t) - kx \tag{A10}$$

where $m$ is the mass of the missile inside the silo, $F(t)$ is the driving force provided by the silo's wall against the spring, and $k$ is the spring's constant related to its stiffness in units of force per length. It is vital to keep in mind that the variable $x$ in this expression is the stretch or compression of the spring. It is not the absolute position of the missile.

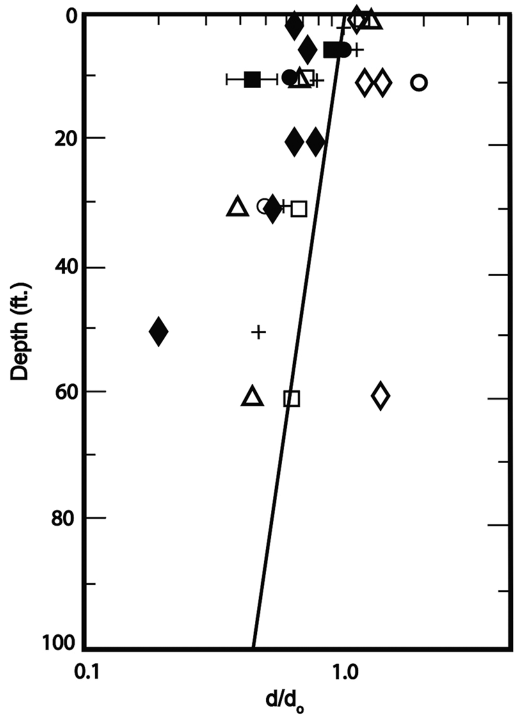


**Figure A4.** Comparison of calculated displacement (in inches) attenuation with normalized peak vertical displacement data from Priscilla Shot. Modified from "Calculation of Vertical Airblast-Induced Ground Motions from Nuclear Explosions in Frenchman Flat," 51.

It needs to be stressed that the idea here is the shock isolation spring's compression should be the same between nuclear and conventional motions. This is related to how the missile experiences acceleration, and it will be destroyed (or at least made unable to launch) if the acceleration its body experiences from a nuclear explosion matches that from a conventional one. This is a simple, plausible way to address the differences between comparable peak ground motions induced by the two explosions.

This effort begins by considering the acceleration, velocity, and displacement of ground motion waveforms over time displayed in Figure A5. Of course with the only reliable data obtained from ground motion data being the peak values of these three parameters, the waveforms in Figure A5 do display sinusoidal behavior. It cannot be claimed that this is exactly the behavior of simple harmonic motion, but the velocity does peak where the acceleration is zero and the ground displacement peaks where the velocity is zero. This later time behavior is really unimportant, however, because the analytical judgment only requires that sinusoidal behavior be present when the silo wall begins to move and that the effect this motion has on the shock isolation system (specifically the spring's compression or whatever attenuating element exists) is the same. The missile might be destroyed early on in the induced motion, but it may also occur later as the silo motion begins to reverse and the shock-attenuated motion allows the missile to experience the maximum acceleration on this return trip and rattle into the opposite silo wall.

Figure A5 displays this sinusoidal behavior in the top plot for the acceleration as the ground begins to move, allowing $F(t)$ to be written as $F\sin(\omega t)$. This allows Equation A10 to be rewritten as

$$\frac{\partial^2 x}{\partial^2 t} + \omega_o^2 x = F\sin(\omega t) \tag{A11}$$

where both sides were divided by $m$ and $\omega_o^2 = \frac{k}{m}$. This type of differential equation is very common with both a homogenous solution $x_h(t)$ when $F\sin(\omega t) = 0$ and particular solution $x_p(t)$ to the equation when it does not. This leads to

$$x_h(t) = C\sin(\omega_o t) + D\cos(\omega_o t) \qquad x_p(t) = \frac{F}{m(\omega^2 - \omega_o^2)}\left[\sin(\omega t)\right] \tag{A12}$$

and when applying the boundary conditions $x(t) = 0$ and $\frac{\partial x}{\partial t} = 0$ for the spring's compression,

$$D = 0 \qquad \text{and} \qquad C = \frac{-F}{m(\omega^2 - \omega_o^2)} \tag{A13}$$

With $x(t) = x_h(t) + x_p(t)$, the final solution for $x(t)$ may be written as

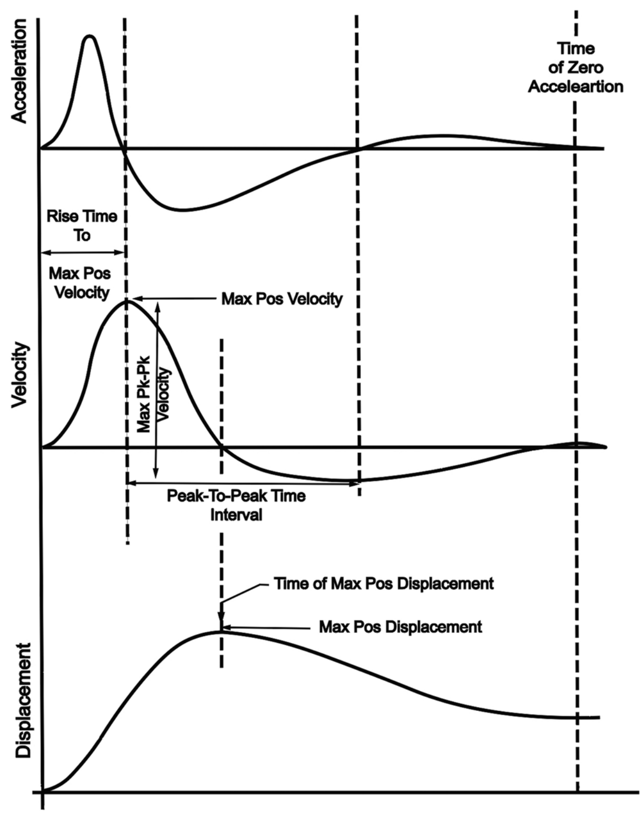


**Figure A5.** The acceleration, velocity, and displacement waveforms over time for ground motion produced by conventional explosions. Modified from C. J. Higgins, R. L. Johnson, and G. E. Triandafilidis, "Simulation of Earthquake-Like Ground Motions with High Explosives, Final Report" (Report CE-45 (78) NSF-507-1 Department of Civil Engineering, University of New Mexico, Albuquerque, NM, 1978).

$$x(t)=\frac{F}{m(\omega^2-\omega_o^2)}\left[\sin(\omega t)-\sin(\omega_o t)\right] \tag{A14}$$

The force $F$ here is really the maximum force of the sinusoidal motion. This should be possible to estimate with the peak motion data available, as this is just the maximum acceleration times the mass $m$ of the silo. The maximum acceleration is just the peak displacement times $\omega^2$. The units do not matter here, as ultimately this is a comparison between nuclear and conventional and units will be common between both. Equation A14 could then be used to set these two motions equal

$$\frac{A_n\omega_n^2 m}{m(\omega_n^2-\omega_o^2)}\left[\sin(\omega_n t)-\sin(\omega_o t)\right]=\frac{B_c\omega_c^2 m}{m(\omega_c^2-\omega_o^2)}\left[\sin(\omega_c t)-\sin(\omega_o t)\right] \tag{A15}$$

where $A_n$ and $B_c$ are the respective nuclear and conventional peak displacements and $\omega_n$ and $\omega_c$ are the angular velocities. With $\sin(\omega t)\approx\omega t$ when $\omega t$ is small, a little more algebra and canceling of common factors leads to

$$\frac{A_n\omega_n^2}{\omega_n+\omega_o}=\frac{B_c\omega_c^2}{\omega_c+\omega_o} \tag{A16}$$

This comparison is becoming clearer, but one additional argument can advance it further. The natural frequency of the shock isolation system combined with the missile is $\omega_o$. By design, this will be much less than the frequency of the ground motion to avoid any resonance that can amplify the missile's motion. In looking at Equation A16, neglecting $\omega_o$ will overstate the magnitude of the nuclear side more than the conventional side given that $\omega_c > \omega_n$. This would once again require conventional weapons to meet an overstated nuclear threshold. This leads to

$$A_n\omega_n = B_c\omega_c \tag{A17}$$

which in terms of peak ground motions requires that the peak parameters of nuclear and conventional be related by

$$\underset{\text{Nuclear}}{\left[\text{peak accel.}\times\text{peak displ.}\right]^{1/2}}=\underset{\text{Conventional}}{\left[\text{peak accel.}\times\text{peak displ.}\right]^{1/2}}$$

This appears to be a solid comparison requiring only that the early response of the shock isolation system be equal between nuclear and conventional ground motions. This expression also equals the velocity, which in theory could provide an even simpler comparison, but using two independently measured parameters is more rigorous and the difficulty with peak velocity data in the nuclear case at depths shallower than 30 ft better supports the use of the relationship displayed here.