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Implosion Nuclear Weapons with 60%-Enriched Uranium

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ABSTRACT

This work considers the direct use of 60%-enriched uranium in pure fission nuclear weapons. From numerical simulations and simple analytics this work shows that substituting a weapons-grade core for a 60%-enriched core reduces the yield by a factor of 4 to 5 that is fairly insensitive to classified design details. A weapon designed to achieve a nominal 10 kt yield using weapons-grade uranium could therefore still produce a modest yield of 2–3 kt using a core of lower enrichment.

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Introduction

While the known Iranian stockpile of 408 kg of uranium enriched to 60% uranium-235 is not canonically "weapons-grade" material, it can certainly still be used to create a nuclear weapon.[1]

With Iranian enrichment capabilities significantly reduced by the US and Israeli airstrikes during the Twelve-Day War in June 2025, this work is motivated to consider what sort of explosive yield could be achieved by a nuclear weapon produced using the existing stockpile without further enrichment. The following sections specifically consider the implications of substituting a highly enriched uranium (HEU) core, nominally 90% ^{235}U or greater, with a core of the same dimensions enriched to only 60% ^{235}U, in a weapon that follows the basic design principles of pure fission weapons.

Readers should treat the calculations in this work as first estimates, as most of the detailed physics for weapon design is not available in the open literature. In the interest of readability, this work omits a detailed discussion of computational methods, preferring instead to make the discussion accessible to a broad audience. Section Implosion weapons discusses the qualitative physics of implosion weapons and Section Numerical methods describes a computational model to calculate yields with the

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FissionBomb code. Section Weapon yields presents fiducial simulations of weapons with 93%-enriched and 60%-enriched cores, as well as a grid of models. Appendix A confirms these simulations analytically.

Implosion weapons

Implosion (or compression) decreases the critical mass of a nuclear weapon core by increasing the number density of fissile nuclei. In the core of an undetonated nuclear weapon under standard conditions the neutron mean free path is relatively long compared to the assembly radius and so neutrons (e.g., emitted by spontaneous fission) are more likely to escape the core than cause additional fissions. Such an uncompressed core is therefore subcritical.

When the material is rapidly compressed by detonating a surrounding shell of conventional explosives, the number density of the fissile nuclei in the core increases. The higher density of potential scattering sites for neutrons reduces the mean free path and decreases the neutron escape probability and by extension the critical mass. While an uncompressed core in an unexploded bomb will be a fraction of its critical mass, a compressed core may contain several multiples of its new critical mass. This abrupt transition from subcriticality to supercriticality, when coupled with a neutron initiator, is the mechanism that initiates the nuclear explosion. It can come as a surprise, even to trained physicists, that a critical mass is not a single constant value. A fixed mass of fissile material may be supercritical in one arrangement and subcritical in another.

The timescale for this compression must be faster than the typical timescale for neutron emission from spontaneous fission to mitigate the risk of predetonation. Such a neutron background can initiate the chain reaction prematurely when the assembly is only barely critical resulting in a "fizzled" explosion with a much lower yield. The spontaneous fission rate of ^{238}U is about three orders of magnitude greater than ^{235}U, so the higher ^{238}U concentration in the lower enrichment considered here merits comment. The spontaneous fission rate for 60%-enriched uranium is $R_{\mathrm{SF}}^{60\%} = 2.72\,\mathrm{kg}^{-1}\mathrm{s}^{-1}$.[2] Assuming Poisson statistics, the probability of observing at least one spontaneous fission in 100 kg of uranium during an assembly time of $\Delta t = 40\,\mu$s is $P(N_{\mathrm{SF}} \geq 1) = \exp(-R_{\mathrm{SF}}\Delta t) \approx 1\%$. This mass and assembly time are both significantly greater than what would be expected in a real weapon; uranium masses of a few tens of kg and assembly times closer to 1 μs would be typical of an implosion device. The risk of predetonation with 60%-enriched uranium is therefore low enough to be neglected from here on.[3]

The simplest design for a fission weapon is shock compression of a solid core with a central neutron initiator, which achieved a compression

of about $\rho/\rho_0 = 2.5$ in the Gadget and Fat Man bombs, where ρ and ρ_0 are the compressed and uncompressed core densities respectively.[4] A theoretical upper limit from shock physics can be shown to be $\rho/\rho_0 = (\gamma+1)/(\gamma-1)$ when the Mach number is large.[5] Taking $\gamma = 5/3$ for a monatomic gas the maximum compression is $\rho/\rho_0 = 4$ though this is energetically prohibitive to realize physically. Substantial gains are possible with a levitated pit and air gap where there is a space left between the core and tamper. While solid core compression is like pressing a hammer against a nail, a levitated pit is like swinging a hammer to hit the nail. When the main charge is detonated, a flyer plate accelerates through the air gap before colliding with the core. It is difficult to estimate the exact compression this achieves, but the maximum may be close to $\rho/\rho_0 \approx 3$ given the relative energetics of shock compression and isentropic compression.[6] This work will be agnostic to the exact method of assembly and will consider compressions over a range.

Numerical methods

The FissionBomb code used here is a fork of the FissionCore code originally developed in a series of papers by Reed[7] and it numerically simulates the time-dependent fission chain reaction using several analytic expressions obtained from diffusion theory. The new version has received updates in Caplan[8] to improve the numerical root finding and to use effective nuclear parameters for isotopic mixtures (i.e., uranium with arbitrary enrichment). For simplicity, FissionBomb does not include full neutron energy spectra or energy dependent cross sections, and uses constant spectrum averaged quantities calculated from the JENDL library by the JAEA.[9] Nuclear parameters used in this work are given in Table 1.

The model assumes a spherical core of mass m_{core} with a surrounding shell tamper of mass m_{tamp}. A small set of fission spectrum averaged nuclear physics parameters as well as the density are the only required inputs. An additional parameter, a compression factor ρ/ρ_0, is used to artificially increase this initial density. This method does not model any collapse and

Table 1. Nuclear physics constants.

	U	Be
A (amu)	236.25	9
ρ (gcm^{-3})	19.05	1.845
σ_{tr} (b)	5.151	2.677
σ_f (b)	0.784	
ν_{eff}	2.555	

Cross sections in barns (1 b = 10^{-24} cm^2). Uranium parameters are obtained from abundance weighted averages of the ^{235}U and ^{238}U parameters with the exception of an artificially lowered σ_f for ^{238}U, discussed in the text; σ_{tr} is a transport coefficient for all scattering.

begins the simulation from this compressed state, taking it as the initial condition for the chain reaction. From this, the time dependent exponential growth in the neutron population and fission reaction rate can be calculated from the number of neutrons $N_n(t) = N_0 \exp(\alpha(t)t)$, where $\alpha(t) = (k-1)/\tau$ is the e-folding timescale for the neutrons, with $k-1$ the neutron multiplication and τ the neutron lifetime against fission. This $\alpha(t)$ is recomputed every timestep from a root finder that determines the critical radius of the core and tamper assembly; the tamper radius and neutron transport mean free path are included in this step to include the neutron reflecting properties of the tamper. The time integrated energy released from the fission chain reaction is used to determine the pressure of a radiation dominated equation of state that drives the time dependent expansion of the assembly. This is coupled to a simple hydrodynamic calculation where the pressure acts on the sum of the core and tamper mass, thereby capturing the confining effects of the tamper. The critical α is therefore dynamically coupled to the hydrodynamic expansion by way of the changing density and neutron mean free paths. At early times, $\alpha(t)$ is effectively constant until the pressure grows large enough to expand the core. Ultimately, the expansion of the core decreases the density allowing neutrons to escape, quenching the chain reaction ($\alpha(t_{\text{final}}) = 0$) at which point the simulation terminates and the final weapon yield is known. For a detailed presentation of the physics interested readers are referred to the works cited above.

This computational approach has several caveats. To start, the detailed interaction of the implosion shocks and outward shock from the explosion are not modeled at all. The yield follows from a simple calculation of the chain reaction with the simplest possible expansion physics, and not from any detailed radiation hydrodynamics. Validation of this model is also challenging, as there are only few data points to calibrate to. Calibration requires the mass and composition of the core and tamper, and yield, to all be publicly known. The most well-documented pure fission weapons in the open literature are the Gadget (Trinity), Fat Man, and Little Boy bombs whose designs have been meticulously reconstructed from declassified sources. This model can reproduce the yields of the Little Boy and Fat Man bombs to order 10%, but with some sensitivity to the exact nuclear physics parameters.[10] Yield estimates given below should therefore be implicitly treated as having a few tens of percent uncertainty. As a highly simplified one-dimensional code FissionBomb makes approximate, not precision, estimates of weapon yields. For the purposes of threat assessment this is quite good, as fireball and overpressure radii scale like $Y^{1/3}$ so even a factor of 2 error in yield would be strongly suppressed. Lastly, FissionBomb is agnostic to details of how the compression is achieved, and therefore may be better thought of as a tool to evaluate the

weapon usability of the fissile material rather than the details of any specific weapon design. As a first order model, it can be especially useful for identifying trends, but readers should be cautious not to over-interpret these results.

Weapon yields

Figure 1 shows time dependent fission burn calculations for weapons with 93%-enriched and 60%-enriched cores. For direct comparison, these simulations both use 20 kg cores surrounded by 25 kg beryllium tampers at compressions of $\rho / \rho_0 = 3$. Qualitatively, these curves are almost identical when scaled appropriately for reasons discussed above. While a 93%-enriched core achieves a yield of roughly 10 kilotons (kt), an equal sized 60%-enriched core still has a modest yield of over 2 kt. In principle this means a pure fission weapon designed for a nominal 10 kt yield whose core has been substituted for one of lower enrichment can still achieve kiloton yields.

There is some uncertainty from nuclear physics, and readers should treat all yields given here as having an uncertainty of order 10%. For the fission cross section of ^{238}U this work uses $\sigma_f^{238U} = 0.10$ b as in Caplan, following the suggestion of Lamoreaux.[11] Increasing this to $\sigma_f^{238U} = 0.15$ b increases the yield at 93%-enrichment (60%-enrichment) to 9.78 kt (2.44 kt).[12] The calculations in Figure 1 also use 100 initiator neutrons, but the calculations are insensitive to the exact choice here so long as it

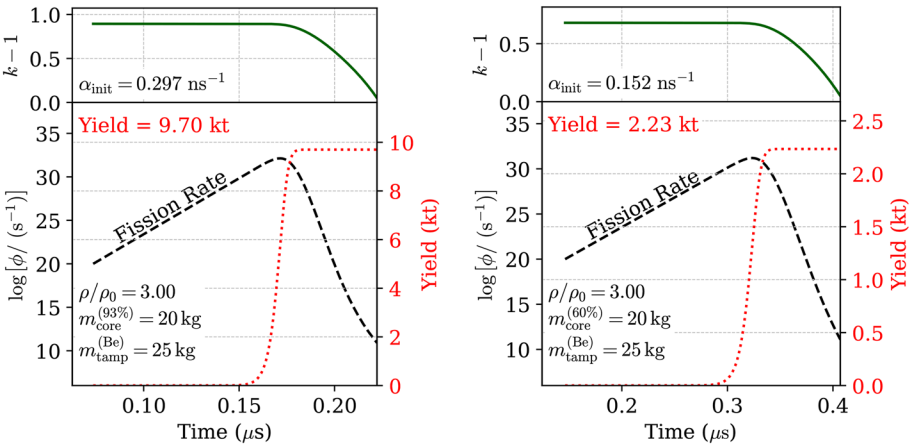

Figure 1. Fiducial weapons with 93%-enriched and 60%-enriched cores. Time dependent fission-burn calculations (bottom) from FissionBomb for identical assemblies show that swapping a 93%-enriched core (left) for a 60%-enriched core (right) will reduce the yield of the device by a factor of about 4. The qualitative explosion physics are the same; the fission rate ϕ grows exponentially due to the exponentially growing number of neutrons. Yields are obtained from the time integrated fission rates and are dominated by the few generations of fissions around peak fission, after which the neutrons begin to escape strongly and the fission quenches. The neutron multiplication (top panels) shows a strong dependence on enrichment, as expected.

is relatively small. This is because the hydrodynamic expansion is driven primarily by the final generations of the fission chain reaction, as the first few e-foldings contribute negligibly to the total energy release and pressure evolution. Varying the initiator neutron population by an order of magnitude up or down increases or decreases the yield by about 5% and is within the error expected here.[13] The choice of 93%-enrichment for the weapons-grade core is also somewhat arbitrary,[14] and yields are highly sensitive to this. Raising or lowering the enrichment of the weapons-grade core by 3% changes the yield by 10% or about a kiloton. For comparison, a 3% change in enrichment at 60%-enrichment has a stronger effect, with almost a 20% change in yield. The choice of a beryllium tamper is reasonable but also speculative, and one might also consider tungsten carbide or another dense material for the tamper or flyer plate, but this is not so important for this analysis.

If the fission rise time is longer than the disassembly time due to the shock rebound then this weapon will not be viable, but 0.3 μs is not so long compared to the assembly timescale to suffer from this. The order of magnitude of the characteristic shock crossing time might be (5 cm) / (8 km/s) $\approx$ 25 μs with 5 cm a typical core radius and 8 km/s a typical upper limit for detonation velocities of conventional explosives. The fission chain reaction is therefore effectively instantaneous relative to any other disassembly time.

The tamper mass and compression can vary inversely at constant yield. As a consequence, there is no way to know from the open literature if the choice of core mass, tamper mass, and compression is representative of a viable weapon design, even if it hits a typical yield and is consistent with some known historical weapons. As a proxy, a grid of plausible tamper masses and compressions are simulated.

Contour plots of yields are given in Figure 2 for models with tamper masses of 10 to 25 kg and core compressions of 2.5 to 3.3 with 20 kg and 25 kg cores. The yield is expected to be strongly dependent on core mass, and these values are taken as upper and lower limits on what a real weapon might use. From Table 1 of Glaser,[15] the bare critical mass of 93%-enriched uranium is given as 53 kg from MCNP calculations, which decreases to 22 kg when surrounded by a 5 cm beryllium tamper (28.5 kg of Be). Note that this means the 25 kg weapons-grade uranium core is above its critical mass when enclosed by a massive close-fitting beryllium tamper, and thus sets the upper limits in mass considered here.

When comparing the 93%-enriched cores (top) to the 60%-enriched cores (center), it is apparent that lowering the enrichment to 60% consistently lowers the yield by a factor of 4–5 (bottom) without much dependence on the exact compression achieved or tamper mass. This is

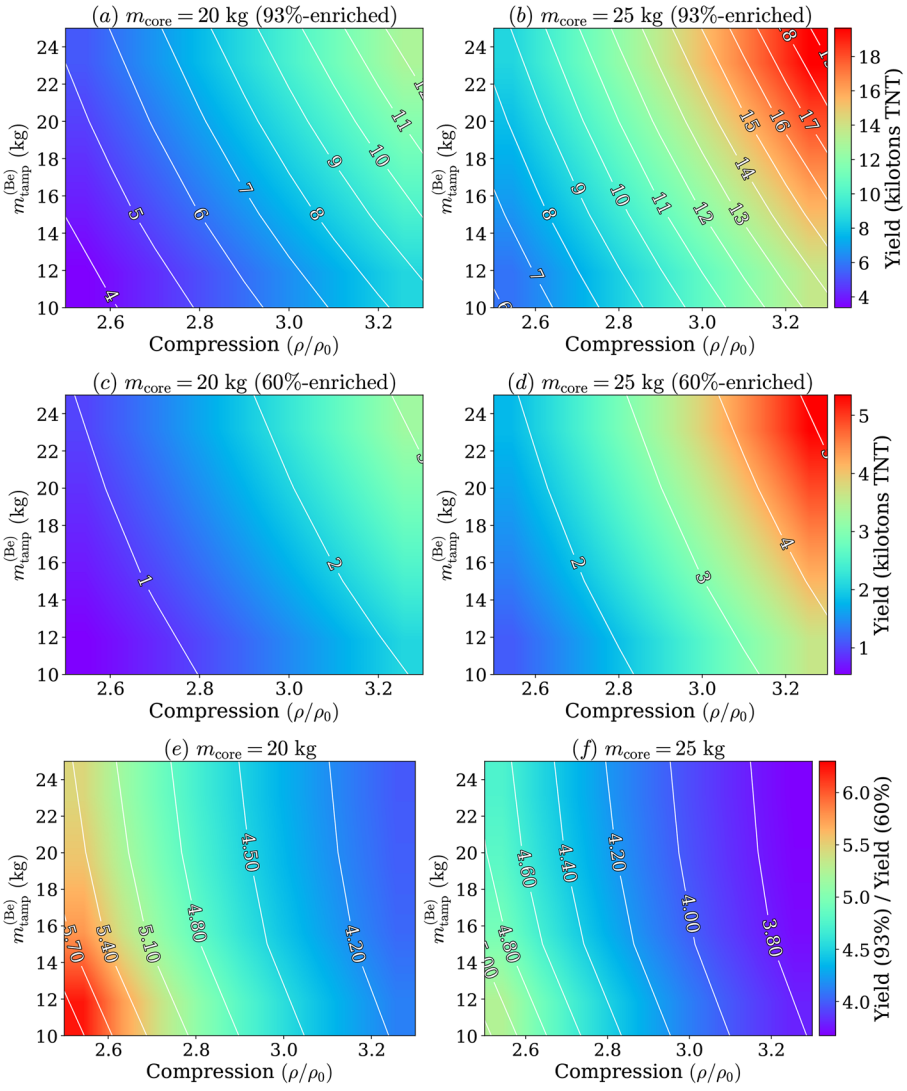

Figure 2. Yield grids: contour plots comparing yields of 20 kg cores (left) and 25 kg cores (right) for 93%-enrichment (top) and 60%-enrichment (center). Reducing the enrichment consistently reduces the yield by a factor of 4 to 5 (bottom), insensitive to the exact design parameters. Note that the absolute values of yields at 93%-enrichment (top) are greater than those at 60%-enrichment (middle), but the underlying gradients are comparable, demonstrating that the yield scales consistently with tamper mass and compression across enrichments. The plots of the reduction factors in (e,f) are just ratios of (a/c) and (b/d).

roughly a result of the universality of the fission burn calculations mentioned above, but the contours are not truly identical between the two enrichments so this reduction is empirical. This can be verified analytically with the yield formula from the Frisch–Peierls memorandum, or the related Bethe-Feynman formula (Appendix A). The added 5 kg of uranium consistently enhances the yield by 1.5× relative to the 20 kg core.

A table of final yields, α_{init}, and times of peak fission are given in Appendix B.

The relatively narrow range of reduction factors can also be seen from other analytic approaches for estimating the yield. In Reed, a simple expression is derived for estimating the yield of a pure fission weapon.[16] This follows from simply treating the fission rate rise as an exponential combined with some expressions for the criticality for determining reaction shutoff. The final expression in Equation (14) in that work gives a yield where $Y \propto M_{tot}$ where M_{tot} is the total assembly mass, i.e., the sum of the core and tamper mass, with the remaining scaling dominated by nuclear physics. Thus, weapons that are identical except for enrichment will have yields in a ratio set largely by the nuclear physics parameters for the enrichment levels under consideration.

Summary

This work has shown that 60%-enriched uranium can be used directly in a nuclear weapon and can achieve kiloton yields with relatively modest design choices. It is apparent that an implosion weapon designed to achieve a nominal 10 kt yield with a weapons-grade uranium core could still achieve a yield of 2–3 kt if the core were swapped for one of 60%-enriched uranium with identical dimensions. While a factor of 4 to 5 lower than yields from weapons-grade cores, these yields are nonetheless significant and should be technically feasible. This reduction is a consequence of the slower neutron multiplication at lower enrichment.

In the absence of further enrichment, this analysis also indirectly shows that there are pathways to weapon designs that can achieve larger yields, perhaps of 10 kt or greater, even if the designer is intending to use 60%-enriched uranium. These pathways involve boosting and gun-type weapons, respectively.

The numerical methods used here are restricted to pure fission weapons and cannot be used to estimate yields with tritium boosting. However, tritium boosting generally requires a fission stage that can reach about 1 kt yields to reach the temperature threshold for effective boosting,[17] so a relatively small amount of tritium can offset the lower level of enrichment in appropriately designed weapon.

Lastly, the importance of the tamper mass to the final yield cannot be disregarded as the reduction in yield due to the lower enrichment can be partially offset with larger tamper masses. The confining effect of a large tamper will retard the expansion of the core, thereby extending the fission burn time and improves the fission efficiency. Even one or two additional neutron generations can raise the yield by several kilotons. While very large tampers may be impractical for missile deliverable implosion weapons, they

could be used to enhance the yield of gun-type weapons delivered by air or in a cargo container.[18] Lower enriched uranium still does not suffer such a serious predetonation risk, as evidenced by the 80%-enriched uranium used in the original Little Boy bomb, or the uranium based gun-type weapons from the South African nuclear weapons program.[19] Because the unreflected critical mass of 60%-enriched uranium is in excess of 100 kg, such a gun-type device could also use a larger mass of uranium in the assembly than the implosion devices with a substituted core considered above.

Notes

1. International Atomic Energy Agency, *IAEA Safeguards Glossary: 2022 Edition* (Vienna: International Atomic Energy Agency, 2022).
2. The SF rate for ^{238}U is 6.78 $kg^{-1}s^{-1}$ from Table 2 in Glaser, "On the Proliferation Potential," such that the composition weighted average at 60%-enrichment is 2.72 $kg^{-1}s^{-1}$. The SF rate for ^{235}U is three orders of magnitude smaller and therefore negligible; Alexander Glaser, "On the Proliferation Potential of Uranium Fuel for Research Reactors at Various Enrichment Levels," *Science & Global Security* 14, no. 1 (2006): 1–24.
3. Historically, this is also why the 80%-enriched uranium available to the Manhattan Project could be used in the gun-type Little Boy with high confidence of success, even despite the much slower assembly time (order milliseconds). The SF rate per mass for ^{238}U is a factor of 70,000 less than that of ^{240}Pu, explaining why uranium weapons do not suffer the same risk of predetonation as plutonium weapons.
4. Thomas M. Semkow, Pravin P. Parekh, and Douglas K. Haines. "Modeling the Effects of the Trinity Test," In *Applied Modeling and Computations in Nuclear Science*, Chapter 11, 142–159 (2006), https://pubs.acs.org/doi/abs/10.1021/bk-2007-0945.ch011.
5. This is just the limiting case of the normal shock equations where $\rho / \rho_0 = [(\gamma+1)M^2] / [(\gamma-1)M^2+2]$ with $M = v_s / c_s$ the Mach number where v_s is the velocity of the shock front and c_s is the speed of sound in the unshocked material.
6. Carey Sublette, "Nuclear Weapons FAQ: Engineering and Design of Nuclear Weapons—Section 4.1," Nuclear Weapon Archive, Version 2.20, 2019 March 13.
7. B. Cameron Reed, "A Brief Primer on Tamped Fission-Bomb Cores," *American Journal of Physics* 77, no. 8 (August 2009): 730–733, https://doi.org/10.1119/1.3125008; Bruce Cameron Reed, "Student-Level Numerical Simulation of Conditions Inside an Exploding Fission-Bomb Core," *Natural Science* 2, no. 3 (2010): 139–144, https://www.scirp.org/journal/paperinformation.aspx?paperid=1559; B. C. Reed and Klaus Rohe, "FISSIONCORE: A Desktop-Computer Simulation of a Fission-Bomb Core," *American Journal of Physics* 82, no. 10 (2014): 972–976.
8. Matt Caplan, "Improvised Nuclear Weapons with 60%-Enriched Uranium," *arXiv* Preprint arXiv:2507.20390 (2025).
9. Tsuneo Nakagawa et al., "Japanese Evaluated Nuclear Data Library Version 3 Revision-2: JENDL-3.2," *Journal of Nuclear Science and Technology* 32, no. 12 (1995): 1259–1271; JAEA, "Nuclear Data Center, Japan Atomic Energy Agency, JENDL-4.0 Internal Data: 92-U-235," 2011 (accessed September 5, 2025).
10. Reed, "Student-Level Numerical Simulation" and Caplan, "Improvised Nuclear Weapons" both contain detailed discussions of Little Boy, which is more relevant given the 80%-enriched uranium used.

11. Steven K. Lamoreaux, "An Elementary Method to Determine the Critical Mass of a Sphere of Fissile Material Based on a Separation of Neutron Transport and Nuclear Reaction Processes," *arXiv* Preprint arXiv:2506.06927 (2025).
12. Both $\sigma_f^{238U} = 0.10\,b$ and $\sigma_f^{238U} = 0.15\,b$ can approximately reproduce the critical mass to within 10% at 60%-enrichment, see Figure 2 in Caplan, "Improvised Nuclear Weapons."
13. Ultimately the number of initiator neutrons will depend on the exact design of the source. For comparison, it is argued that the beryllium/polonium-210 "urchin" initiators used in Fat Man and Gadget produced approximately 100 initiator neutrons when crushed, see B. C. Reed, *The Physics of the Manhattan Project* (Berlin: Springer, 2015).
14. Andrew Brown and Alexander Glaser, "On the Origins and Significance of the Limit Demarcating Low-Enriched Uranium from Highly Enriched Uranium," *Science & Global Security* 24, no. 2 (2016): 131–137.
15. Glaser, "On the Proliferation Potential."
16. B. Cameron Reed, "A Toy Model for the Yield of a Tamped Fission Bomb," *American Journal of Physics* 86, no. 2 (2018): 105–109.
17. In lieu of a detailed physics argument about D-T fusion, it is straightforward to observe that there are several historical examples of variable yield US weapons such as some variants of the W-70 and W-79 warheads that have minimum yields of about 1 kiloton. This yield could be loosely interpreted as the design yield from fission alone, while the higher yield settings make use of the D-T boosting gas, Thomas B. Cochran, William M. Arkin, and Milton M. Hoenig, *Nuclear Weapons Databook, Volume I: US Nuclear Forces and Capabilities* (Natural Resources Defense Council, 1984), 77, https://nuke.fas.org/cochran/nuc_84000001b_01.pdf. More detailed arguments can be found in Carey Sublette, "Nuclear Weapons FAQ: Fission-Fusion Hybrid Weapons—Section 4.3," Nuclear Weapon Archive, Version 2.20, 2019 March 13.
18. Caplan, "Improvised Nuclear Weapons."
19. Reed, *The Physics of the Manhattan Project*; David Albright, *South Africa's Secret Nuclear Weapons* (Institute for Science and International Security, May 1, 1994).
20. Robert Serber, *The Los Alamos Primer: The First Lectures on How to Build an Atomic Bomb* (University of California Press, 1992); John Paul Lestone, M. D. Rosen, and P. Adsley, "Comparison between Historic Nuclear Explosion Yield Formulas," *Nuclear Technology* 207, no. sup1 (2021): S352–S355, https://www.tandfonline.com/doi/full/10.1080/00295450.2021.1960296; B. Cameron Reed, "Revisiting the Frisch–Peierls Memorandum," *The European Physical Journal H* 49, no. 1 (2024): 6, https://link.springer.com/article/10.1140/epjh/s43029-024-00006-x.
21. Walter Seifritz, "A Simple Excursion Model of a Nuclear Explosive Device," *Nuclear Engineering and Design* 239, no. 1 (2009): 80–86.

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Appendix A. Analytic estimates from the Bethe-Feynman yield formula

The Manhattan Project provides useful analytic tools to verify the numeric approach above.[20] Bethe and Feynman presented a yield estimate for a pure fission weapon of

$$Y = f_{\mathrm{BF}} M R_0^2 \alpha_0^2 \delta \tag{A1}$$

where f_{BF} is a prefactor, M is the core mass, R_0 is the radius of the (compressed) assembly when the chain reaction begins, α_0 is the e-folding timescale for neutron growth, and $\delta = R_2 / R_0 - 1$ where R_2 is the radius at "second critical" when the assembly has expanded sufficiently so that the assembly transitions from supercritical to subcritical. There are therefore two relevant compressions, the initial C that sets $R_0 = R_{\mathrm{init}} / C^{1/3}$ from the initial radius R_{init}, and $C_2 < C$ at second critical.

The Bethe-Feynman formula was originally derived for bare spheres without a tamper, requiring justification for its use here. Qualitatively, the Bethe-Feynman formula may be thought of as an analytic estimate of the time integrated fission, especially at early times when α is effectively constant. This is qualitatively not much different from the curves in Figure 1. In principle, with adjusted criticality parameters for reflected criticality and an appropriately chosen prefactor, the Bethe-Feynman should be amenable to tamped cores. Most importantly, the unknown prefactor can be eliminated by considering a ratio of yields.

For identical weapons with substituted cores, the reduction factor can be found from a ratio of yields by writing

$$\frac{Y^{(93\%)}}{Y^{(60\%)}} = \frac{\left[(R_0^2\alpha_0^2)(R_2 / R_0 - 1)\right]^{(93\%)}}{\left[(R_0^2\alpha_0^2)(R_2 / R_0 - 1)\right]^{(60\%)}} \tag{A2}$$

with a superscript denoting the two enrichments. The prefactor and core mass cancel, and so does R_0, but R_0 is retained to use the approximation in Equation (15) of Seifritz,[21]

$$R\alpha = A\left[\rho R - (\rho R)_{\mathrm{crit}}\right] \tag{A3}$$

where A is another prefactor that cancels upon substitution into the ratio

$$\frac{Y^{(93\%)}}{Y^{(60\%)}} = \frac{\left[(\rho_0 R_0 - (\rho R)_{\mathrm{crit}})^2 (R_2 / R_0 - 1)\right]^{(93\%)}}{\left[(\rho_0 R_0 - (\rho R)_{\mathrm{crit}})^2 (R_2 / R_0 - 1)\right]^{(60\%)}}. \tag{A4}$$

It is straightforward to calculate all these radii and densities for a known core mass and knowing the critical masses for both enrichments. When expressed this way, it may be less surprising that a reflected or tamped core can also be treated with the Bethe-Feynman formula, as this equation is simply a statement about the conditions for initial criticality

and second criticality with information about tampers now subsumed into these critical radii.

A power law fit to Table 1 in Glaser suggests the tamped critical mass can be well predicted for enrichments $\varepsilon \gtrsim 0.30$ by $M_{\text{crit}} = (19.3\,\text{kg})\varepsilon^{-1.8} C^{-2}$ and C is the compression. This predicts an uncompressed critical mass at 93% (60%) enrichment of 22 kg (54 kg). The power law drops to −1.7 for a bare core and −1.6 for a 15 cm Be tamper, so a factor of 2.2 difference in critical mass is fairly robust against differences in tamper mass. These critical masses have radii of 6.514 cm and 8.787 cm assuming a density of 19 g/cm^3, giving $(\rho R)_{\text{crit}}^{(93\%)} = 124\,\text{g/cm}^2$ and $(\rho R)_{\text{crit}}^{(60\%)} = 167\,\text{g/cm}^2$, respectively.

The fiducial 20 kg core has an initial radius of $R_{\text{init}} = 6.31\text{cm}$. Under compression $R_0 = R_{init} / C^{1/3}$ such that $R_0 = 4.37\,\text{cm}$ and $\rho_0 = 57\,\text{g/cm}^3$ at $C = 3$ and is the same for both enrichments. The compression at second critical will depend on enrichment and can be found using $C_2 = \sqrt{M_{\text{init}} / M_{\text{crit}}}$, and are $C_2^{(90\%)} = 1.0488 (R_2^{(90\%)} = 6.41\text{cm})$ and $C_2^{(60\%)} = 1.643 (R_2^{(60\%)} = 5.52\,\text{cm})$. Obviously, the 93%-enriched core is very near its critical mass and does not require much compression to begin the chain reaction. The chain reaction at lower enrichment obviously terminates sooner, as second critical is reached with less expansion.

All together this becomes

$$\frac{Y^{(93\%)}}{Y^{(60\%)}} = \frac{\left[(249-124)^2 (6.41/4.37-1)\right]}{\left[(249-167)^2 (5.52/4.37-1)\right]} = 4.3 \tag{A5}$$

in very good agreement with the numerical simulations in Figure 1 and matches very closely with the point just right of center that these parameters correspond to in Figure 2e. The first term corresponds to $\alpha_0^{(93\%)} / \alpha_0^{(60\%)} = 1.53$ differing slightly from the fiducial numerical calculation in Figure 1, $\alpha_0^{(93\%)} / \alpha_0^{(60\%)} = 2.97/1.52 = 1.95$. This is unsurprising, as the analytics are just first estimates, and the error is probably from the cavalier treatment of α and the tamper. It is somewhat remarkable then that the factor of 4.3 is an exact match to Figure 1 given that the Bethe-Feynman expression, like the Frisch-Peierls expression, does not include a tamper. The choice of fitting the power law to the tamped critical mass may partially correct for this, and reporting a ratio in Equation (A4) also likely cancels out other errors that would appear in calculations of the tamped yield alone.

As a final thought, the equations used above can be combined with the power law for the critical mass to obtain an expression for reduction factors for arbitrary core masses, enrichments, and compression factors. This would make it straightforward to estimate the yield of a weapon using lower enriched uranium than the designer intended.

Appendix B. Table of simulation results

This table reports the results of select simulations from the grids in Figure 2 and are formatted with the same layout for ease of reading. For each core mass, tamper mass, and compression factor, the table gives final yields in kilotons, the initial α, and the time of the peak fission rate ϕ_{max}. The fraction of the yield released prior to $t(\phi_{\text{max}})$ is between 58 and 59% of the final yield. As before, readers should implicitly assume a 10% uncertainty for the yields; the additional significant figures are included to aid reproducibility. Notice that varying the tamper mass seemingly does not have a strong effect on α_{init} or $t(\phi_{\text{max}})$, but raises the yield considerably. This is a consequence of the inertial confinement from the tamper and is

Table B1. Each cell lists the column vector $(Y, \alpha_{\text{init}}, t(\phi_{\max}))$, where Y is the yield in kilotons, α is the initial neutron growth timescale, and $t(\phi_{\max})$ is the time of the peak fission rate.

93%-enrichment				
	$m_{\text{core}} = 20\,\text{kg}$		$m_{\text{core}} = 25\,\text{kg}$	
m_{tamp}	$\rho/\rho_0 = 2.5$	$\rho/\rho_0 = 3$	$\rho/\rho_0 = 2.5$	$\rho/\rho_0 = 3$
25 kg	$\begin{pmatrix} 5.23\,\text{kt} \\ 0.213\,\text{ns}^{-1} \\ 0.235\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 9.70\,\text{kt} \\ 0.297\,\text{ns}^{-1} \\ 0.171\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 8.53\,\text{kt} \\ 0.234\,\text{ns}^{-1} \\ 0.216\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 15.03\,\text{kt} \\ 0.320\,\text{ns}^{-1} \\ 0.161\,\mu\text{s} \end{pmatrix}$
10 kg	$\begin{pmatrix} 3.37\,\text{kt} \\ 0.212\,\text{ns}^{-1} \\ 0.234\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 6.39\,\text{kt} \\ 0.296\,\text{ns}^{-1} \\ 0.170\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 5.81\,\text{kt} \\ 0.234\,\text{ns}^{-1} \\ 0.215\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 10.41\,\text{kt} \\ 0.319\,\text{ns}^{-1} \\ 0.160\,\mu\text{s} \end{pmatrix}$
60%-enrichment				
	$m_{\text{core}} = 20\,\text{kg}$		$m_{\text{core}} = 25\,\text{kg}$	
m_{tamp}	$\rho/\rho_0 = 2.5$	$\rho/\rho_0 = 3$	$\rho/\rho_0 = 2.5$	$\rho/\rho_0 = 3$
25 kg	$\begin{pmatrix} 0.96\,\text{kt} \\ 0.100\,\text{ns}^{-1} \\ 0.482\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 2.23\,\text{kt} \\ 0.152\,\text{ns}^{-1} \\ 0.323\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 1.80\,\text{kt} \\ 0.116\,\text{ns}^{-1} \\ 0.420\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 3.81\,\text{kt} \\ 0.170\,\text{ns}^{-1} \\ 0.292\,\mu\text{s} \end{pmatrix}$
10 kg	$\begin{pmatrix} 0.54\,\text{kt} \\ 0.096\,\text{ns}^{-1} \\ 0.492\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 1.39\,\text{kt} \\ 0.150\,\text{ns}^{-1} \\ 0.324\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 1.11\,\text{kt} \\ 0.113\,\text{ns}^{-1} \\ 0.426\,\mu\text{s} \end{pmatrix}$	$\begin{pmatrix} 2.52\,\text{kt} \\ 0.169\,\text{ns}^{-1} \\ 0.292\,\mu\text{s} \end{pmatrix}$

because the product $\alpha t(\phi_{\max})$ sets the number of neutron e-foldings. Consider $t(\phi_{\max})$ as a proxy for the burn time, and notice that the 25 kg tamper generally delays the expansion of the core for an extra few nanoseconds relative to the 10 kg tamper, but this is also comparable to the e-folding timescale set by α. Therefore, the fission burn achieves an additional e-folding with the larger tamper which greatly enhances the reaction efficiency and final yield.

As an explicit example, the lowest yield weapons considered are at 60%-enrichment with a 20 kg core at 2.5 compression and have yields of 0.54 and 0.96 kt. These correspond to $0.482\,\mu\text{s} \times 0.100\,\text{ns}^{-1} = 48.2$ generations of neutrons and $0.492\,\mu\text{s} \times 0.96\,\text{ns}^{-1} = 47.2$. That one additional e-folding is the source of the almost factor of two difference in yield, and makes evident just how important the final few generations of neutrons before shutdown are in determining the total yield.